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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2024/2025ACADEMIC YEAR
FIRST YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS)

Date: 16th April, 2024
Time: 8.30am –10.30am

KMA 105 - DISCRETE MATHEMATICS

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) Let A and B be sets, by use of Venn diagrams show that $\sim(A \cap B) = \sim A \cup \sim B$ (4 marks)

b) Prove that $\sqrt{7}$ is irrational (5 marks)

c) State the truth values of the following:
- *i)* $\forall x \in \mathbb{R}, x + x \geq x$ (1 mark)
- *ii)* $\forall x \in \mathbb{N}, x + x \geq x$ (1 mark)
- *iii)* $\exists x \in \mathbb{N}, 2x + 3 = 6x + 7$ (1 mark)

d) List the members of these sets
- (i) $\{x | 5 \leq x \leq 30$ and x is a sexy prime$\}$ (1 mark)
- (ii) {x| x is a Fibonacci number (2 marks)

e) Write the inverse, converse and contrapositive of the given statement "If you live in Nairobi, then you live in Kenya. (4 marks)

f) Prove that $2 + 4 + 6 + \cdots + 2n = n(n + 1) \forall n \in \mathbb{N}$ (4 marks)

g) Let $A = \{1, a, 2, b, 3, c, 4, d, 5, e\}$, $B = \{2,3,\{a, 1\}, d\}$ and $C = \{3, c, 4,5, b, k\}$. Solve the following:
- i) $P(B)$ (3 marks)
- ii) $A \cup B \cup C$ (1 mark)
- iii) $(A - B) \cap C$ (2 marks)

h) Suppose that *A* is the set of sophomores at your school and *B* is the set of students in discrete mathematics at your school. Express each of these sets in terms of *A* and *B*.
- (i) the set of sophomores taking discrete mathematics in your school (1 mark)
- (ii) the set of sophomores at your school who are not taking discrete mathematics (1 mark)

QUESTION TWO (20 MARKS)

a) State the necessary and sufficient condition of the following conditional statement "if someone is a mother, then she must be a lady". (2 marks)

b) Let $A = \{a, b, c\}$ and $B = \{1,2,3\}$. Compute the cardinality of $A \times B$ and show that $A \times B \neq B \times A$ (4 marks)

c) Construct the truth table of $(p \to q) \wedge [(p \vee \sim r) \to (q \wedge r)]$ (6 marks)

d) Show whether the compound proposition $(p \to q) \leftrightarrow (\sim p \vee q)$ is a tautology (4 marks)

e) Let $f, g: \mathbb{R} \to \mathbb{R}$ be the function with the rule $f(x) = \frac{2}{3}x - 2$ and $g(x) = \frac{5}{2}x + 5$. Find

(i) $f \circ g$ (2 marks)

(ii) $g \circ f$ (2 marks)

QUESTION THREE (20 MARKS)

a) Give one application of discrete mathematics to Information Technology field (1 mark)

b) Test the validity of the argument below:

If $3 - 5 \geq 7$ then either it is rainy or Arsenal will win the league. It is not rainy or Arsenal will not win the league. $3 - 7 \ngeq 7$ if and only if it is not rainy. Therefore if Arsenal will win the league, $3 - 5 \geq 7$. (6 marks)

c) In a survey of 500 people, 285 are interested in football game, 195 are interested in hockey game, 115 are interested in in basketball game, 45 in football and basketball, 70 in football and hockey and 50 in hockey and basketball games and 50 are not interested in any of these three games. By use of a Venn diagram determine:

i) How many people are interested in all the three games? (3 marks)

ii) How many are interested in exactly one of the three games? (2 marks)

iii) How many are interested in at most two of these games? (2 marks)

d) If n is a positive integer, prove that $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$ (6 marks)

QUESTION FOUR (20 MARKS)

a) Distinguish between a tautology and a contingency (2 marks)

b) Use mathematical induction to prove that $9^n - 1$ is divisible by $8, \forall n \in N$. (7 marks)

c) Find the number of integers between 1 and 100 inclusively that are divisible by either 3, 5 or 7 . (6 marks)

d) Let p and q denote: “It is raining”, and “It is wet” respectively. State the verbal translation of each of the following

(i) $p \wedge q$ (1 mark)

(ii) $\neg p \vee q$ (1 mark)

(iii) $\neg p \wedge \neg q$ (1 mark)

(iv) $\neg(p \vee \neg q)$ (1 mark)

(v) $\neg(\neg p \vee \neg q)$ (1 mark)

QUESTION FIVE (20 MARKS)

a) Let *p* and *q* be the propositions *p*: I played in AFCON for the first time.

q: I won the AFCON.

Express proposition $\neg p \vee (p \wedge q)$ as an English sentence. (4 marks)

b) Express the negations of the following propositions using quantifiers and in English

(i) There is a student in this class who has never seen a computer (1 mark)

(ii) Every student in this class likes mathematics (1 mark)

(iii) There is a student in this class who has been in at least one room of every building on campus (1 mark)

c) Given that $f(x) = 2x, g(x) = x^2$ and $\square(x) = x + 1$, find:

(i) $f \circ (g \circ \square)$ (2 marks)

(ii) $g \circ (f \circ \square)$ (2 marks)

d) A survey on a sample of 25 new cars being sold at a local auto dealer was conducted to see which of three popular options, air-conditioning *(A)*, radio *(R)*, and power windows *(W)*, were already installed. The survey found: 15 had air-conditioning *(A)*, 5 had *A* and *W*, 12 had radio *(R)*, 9 had *A* and *R*, 3 had all three options. 11 had power windows *(W)*, 4 had *R* and *W*. Represent this information in a well labeled Venn diagram and hence find the number of cars that had:

i) only *W* (3 marks)

ii) *R* and *W* but not *A* (2 marks)

iii) only one of the options (2 marks)

iv) none of the options (2 marks)