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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS
KMA 303: INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS

Date:

Time:

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) By eliminating the constants, A and B find the differential equation whose solution is;

$y = Ae^{2x} + Be^{-3x}$, (4 Marks)

b) State the Order and the degree of a differential equation;

$$5\left(\frac{d^4 b}{dp^4}\right)^5 + 56\left(\frac{db}{dp}\right)^{10} + b^7 = p$$ (2 Marks)

c) Verify that $y = 3e^{-2x} + 4e^x$ is solution of the equation;

$$\frac{d^3 y}{dx^3} - 3\frac{dy}{dx} + 2y = 0$$ (5 Marks)

d) Using separation of variable technique to solve;

$(x^2+1)y' + y^2 + 1 = 0$ (4 Marks)

e) Determine whether the equation $2xydy + (x^2+y^2)dy = 0$ is exact. Hence, solve it.

(5 Marks)

f) Solve the differential equation using method of variation of parameters.

$$y'' - 2y' + y = \frac{e^x}{x^2}$$ (5 Marks)

g) Radium decays at a rate proportional to the amount present at any time. If the half-life of radium is 6 years, determine the amount present after t years if the initial amount is M_0.

(5 Marks)

QUESTION TWO (20 MARKS)

a) Find a differential equation associated to a circle of radius 3, and centre (a, b). (6 Marks)

b) A copper ball is heated to a temperature of $100^0 C$. Then at time $t = 0$ it is placed in water which is maintained at a temperature of $30^0 C$. At the end of 3 minutes the temperature of the ball is reduced to 70^0 C. Find the time at which the temperature of the ball is reduced to 31^0 C.

(8 Marks)

c) Solve the inhomogeneous differential equation $(x + 2y + 1)dx + (4x + 8y + 6)dy = 0.$

(6 Marks)

QUESTION THREE (20 MARKS)

a) Solve the following differential equation;

$$y' = \frac{x^2 + y^2}{xy}$$

(5 Marks)

b) Find the particular solution of $3xy' - y = Inx + 1$ for $x > 0$satisfying $y(1) = -2$ by the method of integrating factor.

(6 Marks)

c) Find the orthogonal trajectory of the family of curves $x^2 - y^2 = cx$.

(9 Marks)

QUESTION FOUR (20 MARKS)

a) Solve the Bernoulli's differential equation $\frac{dy}{dx} + y = xy^4$

(5 Marks)

b) Find the particular solution of the homogenous differential equation;

$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0$ given that $y(0) = 1,\ y'(0) = -7$ and y left (0 right) =-

(6 Marks)

c) Solve the linear system;

$$x' = -x + 6y$$

$$y' = x - 2y$$

Given $x(0) = 2, y(0) = 0.$

(9 Marks)

QUESTION FIVE (20 MARKS)

a) Find the general solution of the differential equation;

$$(D^5 - D^4 - D^3 - D^2 + 4D - 2)y = 0.$$

(5 Marks)

b) The population of Mwihoko shopping center at any time t given by $N(t)$ is assumed to satisfy the logistic growth law;

$$\frac{dN}{dt} = \frac{1}{800}N(1200 - N)$$

Show that $N(t) = \frac{12000}{1 + ce^{-15t}}$.

(9 Marks)

c) Obtain the general solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x$ using the method of undetermined coefficients.

(6 Marks)