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# KIRIRI WOMEN'S UNIVERSITY OF SCIENCE AND TECHNOLOGY UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR END SEMESTER EXAMINATION FOR THE BACHELOR OF SCIENCE IN COMPUTER SCIENCE KMA 2214: OPERATION RESEARCH METHODS

Date: 14<sup>TH</sup> AUGUST 2023 Time: 11:30AM – 1:30PM

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# **INSTRUCTIONS TO CANDIDATES**

# ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS QUESTION ONE (30 MARKS)

a) What is the procedure for formulating linear programming problem models? (4 Marks)

b) Write the following linear programs in standard form.

$$Min Z = 2x_1 + x_2 + 3x_3$$
 (4 Marks)

Subject to:

$$x_1 + 2x_2 \ge 4$$

$$x_2 - x_3 \le 2$$

$$x_1 + 2x_2 + 3x_3 = 6$$

$$x_1, x_2 \ge 0$$

- c) Distinguish between the following
  - i) Convex and concave set.

(2 Marks)

ii) Feasible solution and optimal

(2 Marks)

d) Consider the following minimizing transportation problem. The supplies at sources 1, 2, and 3 are respectively 10, 10, and 15, and the demands at destinations 1, 2, and 3 are 10, 5, 15. Use the transportation simplex method to find the minimum transportation cost the problem. (6 Marks)

	Destinations				
Origin	1	2	3		
1	9	6	7		
2	5	5	-		
3	8	10	9		

e) Discuss is the significance of post-optimality analysis.

(4 Marks)

f) Solve the problem below and provide all solutions.

$$MaxZ = 6x_1 + 3x_2 - x_3$$
s.t
$$5x_1 + 3x_2 - x_3 \le 100$$

$$4x_1 - 2x_2 \le 10$$

$$x_1 + 3x_2 + x_3 \le 90$$

$$x_1, x_2, x_3 \ge 0$$

#### **QUESTION TWO (20 MARKS)**

a) Solve the linear program graphically, showing the objective function, all constraints, the feasible region. (10 Marks)

$$MinZ = 2x_1 + x_2$$
  
 $s.t$   
 $x_1 + x_2 \ge 4$   
 $x_1 - x_2 \ge 2$   
 $x_1 - 2x_2 \ge -1$   
 $x_1, x_2 \ge 0$ 

b) Use dual simplex method to solve:

$$\begin{aligned} \min Z &= x_1 + 2x_2 + 3x_3\\ \text{Subject to:} & x_1 - x_2 + x_3 \ge 4\\ & x_1 + x_2 + 2x_3 \le 8\\ & x_2 - x_3 \ge 2\\ & x_1, x_2, x_3 \ge 0 \end{aligned} \tag{10 Marks}$$

## **QUESTION THREE (20 MARKS)**

a) A company that manufactures small scale replica toy Ferrari has four factories and three distributors. The demand of the distributors can be met by any of the factories. The table below shows the supply and demand of the toys and the cost to ship a toy from each factory to each distributor.

	D1	D2	D3	SUPPLY
F1	2	5	3	20
F2	2	3	4	30
F3	1	4	5	30
F4	1	3	5	20
DEMAND	35	25	20	

- i) Use the Vogel's Approximation Method to construct the initial basic feasible solution. (4 Marks)
- ii) Perform further iterations to find a transportation schedule that minimizes total transportation cost. (8 Marks)
- b) Discuss the scenario when the following cases occur:

(8 Marks)

- i) Multiple solution
- ii) Degenerate solution
- iii) Unboundiness
- iv) Infeasible

#### **QUESTION FOUR (20 MARKS)**

a) Discuss the five assumptions of a linear programming problem? (10 Marks)

b) Consider the problem.

$$MaxZ = 5x_1 + 7x_2$$
  
 $s.t$   
 $x_1 + 2x_2 \le 4$   
 $x_1 + x_2 \le 3$   
 $x_1, x_2 \ge 0$ 

i) Write the dual to this linear program. (4 Marks)

ii) Find the solution of the dual program from the solving the primal program. (6 Marks)

### **QUESTION FIVE (20 MARKS)**

- a) Prove that a basic feasible solution to a linear program must correspond to an extreme point of the set of all feasible solution. (6 Marks)
- b) Consider the following linear program.

Max 
$$Z = 2x_1 + 4x_2$$
  
Subject to:  $4x_1 + 6x_2 \le 120$   
 $x_2 \le 10$   
 $2x_1 + 6x_2 \le 72$   
 $x_1, x_2, \ge 0$ 

Slack variables  $x_3, x_4, x_5$ , are introduced to the three constraints, and the resulting optimal simplex tableau is.

Basic	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	solution
$x_5$	0	0	1/6	-1/6	1	6
$x_1$	1	0	1/2		0	24
$x_2$	0	0	-1/6	1/6	0	4
Z	0	0	1/3	2/3	0	64

- i) What is the range of values of the right hand side of the first constraint for which the dual price is valid? (3 Marks)
- ii) Assume the right hand side of constraint 2 is allowed to change to 10 + S. Find the range for S that retains the optimal bases solution. (4 Marks)
- iii) What is the shadow price for each constraint? (3 Marks)
- iv) Give the range of optimality for the unit profit for first decision variable. (4 Marks)