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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2016/2017 ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

Date: 15th August, 2016.
Time: 8.30am – 10.30am

KMA 203 – PROBABILITY AND STATISTICS II

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) State the following theorems;

i) Chebyshev Inequality .

(2 Marks)

ii) Laws of Large Numbers.

(2 Marks)

b) Let X have the probability density function;

$$f(x)=\begin{cases}\dfrac{x^2}{9} & 0<x<3\\ 0 & elsewhere\end{cases}$$

Find the probability density function of $Y=X^3$ using the cumulative distribution function technique.

(5 Marks)

c) Consider the joint probability distribution given by;

$$f(x,y)=\begin{cases}\dfrac{x+2y}{18}, & x=1,2;\ y=1,2\\ 0, & elsewhere\end{cases}$$

i) Construct the joint probability distribution table of X and Y.

(5 Marks)

ii) Find $P[(X,Y)\in A]=P(X+Y\le 3)$

(4 Marks)

d) Let X and Y be two independent random variables with identical probability density function given by;

$$f(x)=\begin{cases} e^{-x} & for\ x>0 \\ 0 & elsewhere \end{cases}$$

What is the probability density function of $W=\max\{X,Y\}$?

(6 Marks)

e) If $X \sim F(3,8)$, find the mean and variance of X.

(6 Marks)

<u>QUESTION TWO (20 MARKS)</u>

a) Derive the moment generating function of a random variable X whose probability mass function is the binomial distribution with parameters n and p. Hence or otherwise derive the mean and variance of X using the moment generating function obtained.

(10 Marks)

b) Let X and Y have the bivariate density function;

$$f(x,y)=\begin{cases} \frac{1}{2}xy, & 0<x<y<4 \\ 0, & \text{elsewhere} \end{cases}$$

Determine;

i) The conditional density of Y given $X=x$

(5 Marks)

ii) $E[Y/X=x]$

(5 Marks)

<u>QUESTION THREE (20 MARKS)</u>

a) State the Central Limit Theorem.

(2 Marks)

b) Let $\overline{X}$ denote the mean of a random sample of size n = 15 from the distribution whose probability distribution function is;

$$f(x)=\begin{cases} \frac{3}{2}x^2 & if\ -1<x<1 \\ 0 & otherwise \end{cases}$$

Find μ and σ^2. Hence compute the $P(0.05 \le \overline{X} \le 0.15)$

(8 Marks)

c) If X is an F-distributed random variable with m and n degrees of freedom. Show that $E[X]=\frac{n}{n-2}$ $for\ n>2$. Hence if $X \sim F(9,10)$, find the mean of X

(10 Marks)

QUESTION FOUR (20 MARKS)

a) Given the probability distribution function of a Gamma distribution as;

$$f(x) = \begin{cases} \dfrac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta}, 0 < x < \infty \\ 0, elsewhere \end{cases}$$

Obtain;

i) Moment generating function (10 Marks)

ii) Mean and variance (5 Marks)

b) Let X have a gamma distribution (given in part (a)) with $\beta = 2$ and $\alpha = r/2$, where r is a positive integer.

i) Write down the probability distribution function of X. (3 Marks)

ii) Mean and Variance. (2 Marks)

QUESTION FIVE (20 MARKS)

The joint probability of density of X and Y is;

$$f(x,y) = \begin{cases} k(5x + y), & 0 < x < 1,\ 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

a) Find the value of k. (3 Marks)

b) Find;

i) $E[X]$ (2 Marks)

ii) $E[Y]$ (2 Marks)

iii) $E[XY]$ (3 Marks)

iv) $Var(X)$ (3 Marks)

v) $Var(Y)$ (3 Marks)

c) Correlation coefficient between X and Y. (4 Marks)