



Kasarani Campus  
Off Thika Road  
Tel. 2042692 / 3  
P. O. Box 49274, 00100  
NAIROBI  
Westlands Campus  
Pamstech House  
Woodvale Grove  
Tel. 4442212  
Fax: 4444175

**KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY**  
UNIVERSITY EXAMINATION, 2020/2021 ACADEMIC YEAR  
FIRST YEAR, SECOND SEMESTER EXAMINATION  
FOR THE DEGREE OF BACHELOR OF EDUCATION  
(ARTS)

Date: 10<sup>th</sup> December, 2020  
Time: 11.30am – 1.30pm

**KMA 2104 - LINEAR ALGEBRA 1**

**INSTRUCTIONS TO CANDIDATES**

**ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS**

**QUESTION ONE (30 MARKS)**

- a) By reducing the given system of linear equations to its echelon form, determine the values of  $x$ ,  $y$  and  $z$

$$2x + y + z = 1$$

$$-x + 2y - 3z = 3$$

$$x + 3y - 2z = 4$$

(5 Marks)

- b) Is  $-1 + x^2$  in the span of  $p = 1 + x + x^3$  and  $q = -x - x^2 - x^3$  in  $P_3$ ?

(4 Marks)

- c) Show that  $w = \{(x, y, 2) : x, y \in R\}$  is not a subspace for  $R^3$ .

(2 Marks)

- d) Find the values of  $k$  for which the matrix  $T = \begin{bmatrix} k-3 & 4 \\ k & k+2 \end{bmatrix}$  has no inverse. (5 Marks)

- e) Determine if  $b$  is a linear combination of the vectors

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}, u_3 = \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}, \text{ where } b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}.$$

(5 Marks)

- f) Determine whether the set of vectors  $\{(3, 1, 1), (2, -1, 5), (4, 0, -3)\}$  is linearly dependent in  $IR^3$ .

(4 Marks)

- g) Prove that  $(AB)^{-1} = B^{-1}A^{-1}$  and hence verify using the matrices

$$A = \begin{bmatrix} 1 & -3 \\ 0 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}$$

(5 Marks)

**QUESTION TWO (20 MARKS)**

a) Is the set  $T = \{(1, 1, 1), (2, 1, -1), (1, 0, -2)\}$  a basis for  $\mathbb{R}^3$ ? (6 Marks)

b) Solve the linear system

$$5x_1 - 2x_2 + x_3 = 1$$

$$3x_1 - 2x_2 = 3$$

$$x_1 + x_2 - x_3 = 0$$

using Cramer's rule. (6 Marks)

b) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$$

by cofactor expansion method. (8 Marks)

**QUESTION THREE (20 MARKS)**

a) Determine the dimension of and a basis for the solution space for the system

$$x + y + z = 0$$

$$3x + 2y - 2z = 0$$

$$4x + 3y - z = 0$$

$$6x + 5y + z = 0$$

(9 Marks)

b) Determine whether the set  $S = \{2 + x + x^2, x - 2x^2, 2 + 3x - x^2\}$  is linearly independent in  $P_2$ .

(6 Marks)

c) Find the value of  $k$  for which the following matrix  $A$  is singular.

$$A = \begin{bmatrix} 1 & 2 & k \\ 3 & -1 & 1 \\ 5 & 3 & 5 \end{bmatrix}$$

(5 Marks)

**QUESTION FOUR (20 MARKS)**

- a) Determine the value of 'a' so that

$$\begin{aligned}x_1 - 3x_3 &= -3 \\2x_1 + ax_2 - x_3 &= -2 \\x_1 + 2x_2 + ax_3 &= 1\end{aligned}$$

has

- i) No solution (4 Marks)  
ii) Unique solution (2 Marks)  
iii) Many solution. (2 Marks)

- b) Solve using Cramer's rule

$$\begin{aligned}2x + y - 2z &= 10 \\3x + 2y + 3z &= 1 \\5x + 4y + 3z &= 4\end{aligned}$$

(5 Marks)

- c) Prove that if A is an  $n \times n$  matrix which is invertible, then for any vector  $\underline{b}$  in  $R^n$  the linear system  $A\underline{x} = \underline{b}$  has a unique solution  $A^{-1}\underline{b}$ . Hence find a unique solution for the system

$$\begin{aligned}3x + 2y + 3z &= 1 \\2x - 2y + 4z &= 6 \\4x + 5y - z &= -2\end{aligned}$$

(7 Marks)

**QUESTION FIVE (20 MARKS)**

- a) Express the vector  $\underline{u} = (0, 1, 2)$  as a linear combination of the vectors  $\underline{v}_1 = (-1, 1, 0)$ ,  $\underline{v}_2 = (2, 0, 1)$ ,  $\underline{v}_3 = (1, 1, 1)$ .

(7 Marks)

- b) Show that  $S = \{(1, 2, 1), (2, 9, 0), (3, 3, 4)\}$  is a basis for  $R^3$ .

(7 Marks)

- c) Find the basis and dimension of the solution space for the equations

$$\begin{aligned}2x_1 + 2x_2 - x_3 + x_5 &= 0 \\-x_1 - x_2 + 2x_3 - 3x_4 + x_5 &= 0 \\x_1 + x_2 - 2x_3 - x_5 &= 0 \\x_3 + x_4 + x_5 &= 0\end{aligned}$$

(6 Marks)