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**KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY**  
**UNIVERSITY EXAMINATION, 2020/2021 ACADEMIC YEAR**  
**FIRST YEAR, SECOND SEMESTER EXAMINATION**  
**FOR THE DEGREE OF BACHELOR OF SCIENCE**  
**(MATHEMATICS)**

Date: 18<sup>th</sup> December, 2020  
Time: 11.30am – 1.30pm

**KMA 105 - DISCRETE MATHEMATICS**

**INSTRUCTIONS TO CANDIDATES**

**ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS**

**QUESTION ONE (30 MARKS)**

- a) List the members of the set  $A = \{x \mid x \in \mathbb{Z}, x = 3r, r \in \mathbb{Z} \text{ and } 0 \leq r \leq 4\}$   
(2 Marks)
- b) Let  $A = \{1, 2, 3, 4, 5\}$  and  $B = \{0, 3, 6\}$ . Find i)  $A \cup B$ , ii)  $A \cap B$ , iii)  $A - B$ , iv)  $B - A$ .  
(4 Marks)
- c) If  $A = \{a, b, c, d\}$  and  $B = \{1, 2, 3\}$  find  $A \times B$   
(4 Marks)
- d) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  with  $f(x) = x^2$ . Determine whether  $f$  is one-to-one and onto.  
(4 Marks)
- e) Prove that if  $n$  is an integer and  $3n + 2$  is odd, then  $n$  is odd using contraposition method.  
(3 Marks)
- f) Write the converse, inverse and contrapositive of the following statement "If Maria learns Discrete Mathematics, then she will find a job."  
(6 Mark)
- g) Prove that  $\sqrt{p}$  is irrational for any prime  $p$ .  
(4 Marks)
- h) Let  $p$  and  $q$  be the propositions  
 $p$ : I bought a lottery ticket this week.  
 $q$ : I won the million Shillings jackpot.  
Express proposition  $\neg p \vee (p \wedge q)$  as an English sentence.  
(3 Marks)

### QUESTION TWO (20 MARKS)

- a) Out of a group of 85 people, 30 invested in the stock market, 45 had certificates of deposits (CD's) and 44 had saving bonds. Furthermore, 23 had both CD's and bonds, 13 had both CD's and stocks and 13 had stocks and bonds. Finally 10 of the people had no investment. Represent this information in a well labelled Venn diagram and hence find the number of people that had:
- i) All the three types of investments (4 Marks)
  - ii) At least two investments. (3 Marks)
  - iii) At most two investment (3 Marks)
  - iv) Saving bonds only (3 Marks)
  - v) CD's only (3 Marks)
- b) Show that  $(p \rightarrow r) \wedge (q \rightarrow r)$  and  $(p \vee q) \rightarrow r$  are logically equivalent (4 Marks)

### QUESTION THREE (20 MARKS)

- a) Use a direct proof to show that if  $n$  is an even integer, then 4 divides  $n^2$  (4 Marks)
- b) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = 2x - 3$ . Determine whether  $f$  is invertible and if so find  $f^{-1}$  (4 Marks)
- c) Find the power set  $P(A)$  of  $A = \{a, b, c, d\}$ . (4 Marks)
- d) Using set builder notation show that  $\overline{A \cup B} = \bar{A} \cap \bar{B}$ , if  $A$  and  $B$  are sets (4 Marks)
- e) Prove using mathematical induction that  $1 + 2 + 3 + \dots + n = \frac{1}{2}n(n + 1)$  (4 Marks)

### QUESTION FOUR (20 MARKS)

- a) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = 2x + 1$  and  $g(x) = x^2 - 2$ . Find the formula for the composition functions  $gof, fog$  and  $fof$  (6 Marks)
- b) Show that for any two sets  $A - B = A \cap B^c$  (3 Marks)
- c) Prove that  $\sqrt{5}$  is irrational by contradiction (7 Marks)
- d) Using a Venn diagram with sets  $A, B$  and  $C$ , shade the following sets;
- i)  $A - (B \cup C)$  (2 Marks)
  - ii)  $\bar{A} \cap (B \cup C)$  (2 Marks)

### QUESTION FIVE (20 MARKS)

- a) Prove that  $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$  (6 Marks)
- b) Prove if  $x$  is odd and  $y$  is even, then  $xy$  is even. (4 Marks)
- c) Prove that the sum of the first  $n$  even natural numbers is  $n(n + 1)$ . (6 Marks)
- d) Prove that if  $A \times B \neq \emptyset$  and  $A \times B = C \times D$  then  $A = C$  and  $B = D$ . (4 Marks)