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KIRIRI WOMEN'S UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2023/2024 ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

KMA 205 - BASIC NUMBER THEORY

Date: 15th December 2021
Time: 11.30m – 1.30pm

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Explain with examples the difference between the following terms as used in number theory

(i) Prime numbers and composite numbers

(ii) Rational numbers and integers. (8 Marks)

b) Show that if $\frac{d}{a}$ and $\frac{d}{b}$, then $\frac{d}{ra \pm sb}$ (4 Marks)

c) Prove that every composite integer n has a prime divisor p such that $1 < p < \sqrt{n},$ hence if an integer n has no prime divisor between 1 and $\sqrt{n},$ then n must be prime. (4 Marks)

d) For positive integers 485 and 625, show that $(485, 625) = 5$ (4 Marks)

e) Prove that if $\frac{n}{ab}$ and n and a are coprime, then $\frac{n}{b}$ (5 Marks)

f) State the Wilson's theorem (2 Marks)

g) Solve for $x^2 + y^2 \cong 0(\text{mod}\,3)$ (3 Marks)

QUESTION TWO (20 MARKS)

a) Find all the right-angled triangles with integer sides and a perimeter of 240 (12 Marks)

b) Show that $(723{,}387) = 3$ and find values of x and y such that $723x + 387y = 3$ (8 Marks)

QUESTION THREE (20 MARKS)

a) If $a \cong b(\bmod m)$ and $c \cong d(\bmod m)$, show that $a \pm c \cong b \pm d(\bmod m)$ (4 Marks)

b) Solve $x \cong 4(\bmod 21)$ and $x \cong 13(\bmod 30)$ simultaneously (8 Marks)

c) Find the solutions of the linear Diophantine equation $109x + 87y = 50001$ (8 Marks)

QUESTION FOUR (20 MARKS)

a) Define pseudo-prime (2 Marks)

b) State the Fermat's theorem hence find the order of $2(\bmod 167)$ (12 Marks)

c) Prove that if $(a,b) = 1$, the equation $ax + by = c$ can be solved in integers. If x_0, y_0 is one of the solution, then the general solution is $x = x_0 + bt,\ y = y_0 - at$ where t is an arbitrary integer.

(6 Marks)

QUESTION FIVE (20 MARKS)

a) State Helly's theorem (4 Marks)

b) Solve $3x - 5y + 7z = 12,\ \ 5x + 9y - 11z = 40$ by eliminating z and solve the linear Diophantine equation obtained. (12 Marks)

c) Show that $\sqrt{689}$ is a prime number (4 Marks)