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**KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY**  
**UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR**  
**FIRST YEAR, FIRST SEMESTER EXAMINATION**  
**FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS)**

Date: 7<sup>th</sup> December, 2023  
Time: 8.30am –10.30am

**KMA 2102 - DIFFERENTIAL CALCULUS**

**INSTRUCTIONS TO CANDIDATES**

**ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS**

**QUESTION ONE (30 MARKS)**

- a) Evaluate the limits
- i)  $\lim_{x \rightarrow 1} \frac{x-2}{x^2+x-6}$  (2 marks)
- ii)  $\lim_{x \rightarrow \infty} \frac{2x^4 - x^2 + 8x}{-5x^4 + 7}$  (2 marks)
- iii)  $\lim_{x \rightarrow 0} \frac{\log(1+x)}{x}$  (2 marks)
- b) Differentiate  $f(x)$  from first principles given  $f(x) = -2x^2 + 3$  (3 marks)
- c) A particle moves along a straight line so that its displacement,  $S$  metres, from a fixed point in it, after  $t$  seconds is given by  $s(t) = 6t^2 - t^3$ .
- i) Find the time after the start when the velocity is zero. (2 marks)
- ii) At what time after the start is the acceleration zero? (2 marks)
- d) What value must be assigned to  $k$  so that the following function is continuous at  $x = 4$
- $$f(x) = \begin{cases} \frac{x^2-25}{x-5}, & x \neq 5 \\ k, & x = 5. \end{cases} \quad (3 \text{ marks})$$
- e) Given  $f(x) = 3x - 2$  and  $h(x) = x^3$ , find each of the following:
- i)  $(f \circ h)(x)$  (2 marks)
- ii)  $(h \circ f)(x)$  (2 marks)
- f) Given that  $f(x) = \frac{x+4}{2x-5}$ , find  $f^{-1}(x)$ , the inverse of  $f(x)$ . (2 marks)
- g) Differentiate the following functions:
- i)  $y = x^2 - \frac{1}{2x^3}$ . (3 marks)
- ii)  $y = \tan(3x^2 + 2x + 5)$ . (3 marks)
- iii)  $y = e^x(2x^2 + 1)$  (2 marks)

### **QUESTION TWO (20 MARKS)**

- a) Find  $\frac{dy}{dx}$  for the following
- $y = \ln(x^2 + 3x + 1)$  (3 marks)
  - $y = \frac{3x^4}{(x-1)^4}$  (3 marks)
  - $y = \frac{1+\cos x}{1-\cos x}$  (3 marks)
- b) Find domain and range for each of the following functions.
- $f(x) = x^2 + 4x - 5$  (2 marks)
  - $f(x) = \sqrt{x+2} - 4$  (2 marks)
  - $f(x) = \frac{5}{x-3}$  (2 marks)
- c) Consider the curve  $y^2 + xy - x^2 = 7$ . Find the equation of the tangent line to the curve at the point  $(2, 3)$ . (3 marks)

### **QUESTION THREE (20 MARKS)**

- a) Differentiate the following function implicitly
- $y^2 + x^3 - y^3 + 6 = 3y$  (3 marks)
  - $y^2 + x^3 - xy + \cos y = 0$  (3 marks)
- b) Differentiate the following functions:
- $f(x) = e^{x^2}(2x^2 + 3x)$  (2 marks)
  - $y = \ln(x^2 + 2)$  (2 marks)
  - $f(x) = e^{3x} \sin(2x + 1)$  (2 marks)
  - $y = \left(\frac{x}{3x+2}\right)^5$  (2 marks)
  - $y = 5x^3 \sin x$  (2 marks)
  - $f(x) = \frac{\cos x}{4x^2}$  (2 marks)
  - $y = \cos^4(7x^2 + 1)$  (2 marks)

### **QUESTION FOUR (20 MARKS)**

- a) Find the equation of the line tangent to the given curve  $x = 2t - 1, y = 4t^2 - 2t$  at the point  $t=1$ . (4 marks)
- b) Show that  $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$  (5 marks)
- c) A spherical balloon is being filled with air at the constant rate of  $2 \text{ cm}^3/\text{sec}$ . How fast is the radius increasing when the radius is 3 cm? (5 marks)

d) Find the derivatives of the following functions

i)  $y = \frac{\sin x}{x}$  (2 marks)

ii)  $y = \sin x \cos x$  (2 marks)

iii)  $y = 5e^x + \cos x$  (2 marks)

**QUESTION FIVE (20 MARKS)**

a) Given the following function  $f(x) = -x^3 + x^2 + 8 - 12 - 4x^3$ .

i) Find the x-intercepts of the graph f. (3 marks)

ii) Calculate the turning points of the given function (3 Marks)

iii) Sketch the graph (2 Marks)

b) Show that  $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3} = -\frac{1}{6}$ .

(3 Marks)

c) A soccer ball is kicked into the air so that the path of its flight can be modeled by the function  $x(t) = -4.9 t^2 + 9.8t + 5$  where t is in seconds and x is meters above the ground.

i) At what time will the ball land?

(2 marks)

ii) How many meters above the ground was the ball kicked?

( 2 marks)

iii) Find the maximum height the ball will reach and at what time will this occur?

(3 marks)

iv) What is the acceleration of the ball at  $t = 3$  s?

(2 Marks)