Kasarani Campus
Off Thika Road
Tel. 2042692 / 3
P. O. Box
49274, 00100
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS
KMA 209: ALGEBRA 1

Date:
Time:

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) List three types of groups. Explain why the set of integers under multiplication is not a group. (5 Marks)

b) Distinguish between a symmetric group and alternating group. (4 Marks)

c) Explain the meaning of a subgroup, list all the subgroups of Z_6 and construct their lattice diagram. (5 Marks)

d) Prove that for all $a,b\in G$ then $(ab)^{-1}=b^{-1}a^{-1}$ (3 Marks)

e) Define ¿ on Q^{+} by $a*b=\frac{2ab}{3}$, show that ¿¿ is a group. (4 Marks)

f) Let m be a fixed positive integer in Z. Define the relation $¿_n$ on Z by $x\equiv_n y$ iff $\frac{n}{x-y}$ for all $x,y\in Z$. Show that $¿_n$ is an equivalence relation in Z. (4 Marks)

g) Show that every division ring is a ring without zero divisor. (5 Marks)

QUESTION TWO (20 MARKS)

a) Let G denote the set of all ordered pairs of real numbers with non-zero first component of the binary operation ¿ is defined by $(a,b)*(c,d)=(ac,bc+cd)$. Show that ¿¿ is a non-abelian group. (6 Marks)

b) Let A be a non-empty set and let S_A be the collection of all permutations of A. Then S_A is a group under permutation multiplication. (7 Marks)

c) Prove that every cyclic subgroup is abelian hence show how 1 generates Z_{12} (7 Marks)

QUESTION THREE (20 MARKS)

a) Define an integral domain. (2 Marks)

b) In an integral domain, if $ac=ad$ then $c=d$ if $a\neq 0$ (4 Marks)

c) Let $f: G \rightarrow G_1$ be a group homomorphism. Show that the kernel of f is a normal subgroup of G.
(8 Marks)

d) Show that every subgroup of index 2 is normal. (3 Marks)

e) Show that any two cosets, left of right cosets of subgroup H of group G are disjoint.
(3 Marks)

QUESTION FOUR (20 MARKS)

a) Define normal subgroup show that every subgroup of an abelian group is normal.
(6 Marks)

b) Let H be a normal subgroup of $G.$ Denote the set of all left cosets $\{aH | a \in G\}$ by $\frac{G}{H}$ and define * in $\frac{G}{H}$ for all $aH, bH \in \frac{G}{H}$ by $(aH)*(bH) = abH.$ Show that ¿¿ is a group.
(8 Marks)

c) Let R_1 and R_2 be subrings of R .Show that $R_1 \cap R_2$ is a subring of $R.$
(6 Marks)

QUESTION FIVE (20 MARKS)

a) State the Langrange's Theorem. (3 Marks)

b) List all the elements of a symmetric group of order 3 and construct a multiplication table for S_3
(12 Marks)

c) Prove that any two cosets; right and left cosets of H in G are disjoint. (5 Marks)