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# KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2024/2025ACADEMIC YEAR
THIRD YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS)

Date: 16th April, 2024
Time: 8.30am –10.30am

## KMA 302 - COMPLEX ANALYSIS 1

**INSTRUCTIONS TO CANDIDATES**

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

**QUESTION ONE (30MARKS)**

**a)** If $z_1 = 2 + i,\ z_2 = 3 + 2i,\ and\ z_3 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i,$ evaluate

i. $|3z_1 - 4z_2|$ [3 Marks]

ii. $(\overline{z_3})^4$ [3 Marks]

**b)** Prove that $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$ [2 Marks]

**c)** Express $2 + 2\sqrt{3}i$ in polar form [3 Marks]

**d)** Find the Laurent's series expansion of the function

$$f(z) = \frac{1}{1-z}$$ [4 Marks]

**e)** Solve for z in the following equation

$$z^4 + 81 = 0$$ [3 Marks]

**f)** Show that $\cos 5\theta = 11\cos^5\theta - 15\cos^3\theta + 5\cos\theta$ [4 Marks]

**g)** Determine the residue of the following function at each of the poles

$$f(z) = \frac{z^2}{(z-1)^2(z+2)}$$ [4 Marks]

**h)** Apply Cauchy-Residue theorem to evaluate

$$f(z) = \frac{2z-1}{Z(Z+1)(Z-3)}dz \quad \text{where c is a circle } |z| = 2.$$ [4 Marks]

## QUESTION TWO (20 MARKS)

**a)** Let $u(x,y) = e^{-x}(x \sin y - y \cos y)$

(i) Show that $u(x,y)$ is harmonic [4 Marks]

(ii) Find $v(x,y)$ such that $f(x,y) = u(x,y) + v(x,y)$ is analytic [4 Marks]

**b)** Use Residue theorem to evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta} d\theta$ [8 Marks]

**c)** Show that $\sin^5 \theta = \frac{1}{16}(\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta)$ [4 Marks]

## QUESTION THREE (20 MARKS)

**a)** Let $f(z) = \frac{z^2+4}{z-2i}$ if $z \neq 2i$, while $f(2i) = 3 + 4i$

(i) Prove that $\lim_{z \to 2i} f(z)$ exists and determine its value. [3 Marks]

(ii) Is $f(z)$continuous at $z = 2i$? Explain. [3 Marks]

(iii) Is $f(z)$continuous at points $z \neq 2i$? Explain. [3 Marks]

**b)** Use Cauchy's Integral Formula to evaluate

$\int_C \frac{e^{2z}\, dz}{(z+1)^4}$ where C is the circle$|z| = 3$ [6 Marks]

**c)** Find the Laurent series of the function $\frac{e^{2z}}{(z-1)^3}$ about the singularity $= 1$ .

Name the singularity and give the region of convergence. [5 Marks]

## QUESTION FOUR (20 MARKS)

**a)** Write down $z$ given that $|z| = 2$ and $\arg z = 150^\circ$ [3 Marks]

**b)** Prove that a necessary condition that $f(z) = u(x,y) + iv(x,y)$ to be analytic in the region D is that the Cauchy Riemann equations $\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = -i\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}$. Hence show that $f(z) = 2z^2 + 5$ is analytic. [8 Marks]

**c)** Simplify $\frac{(\cos 3\theta + i \sin 3\theta)(\cos 5\theta - i \sin 5\theta)}{(\cos 2\theta - i \sin 2\theta)(\cos 7\theta + i \sin 7\theta)}$ [4 Marks]

**d)** Prove that $\sin z = \sin x \cosh y - i \cos x \sinh y$ [5 Marks]

**QUESTION FIVE (20 MARKS)**

a) Use Cauchy's Residue Theorem to evaluate the real integral

$$\int_{-\infty}^{+\infty} \frac{1}{(x^2+1)(x^2+3)}\,dx$$ [6 Marks]

b) Evaluate $\int_{3i}^{2+4i} (2y + x^2)dx + (3x - y)dy$ along the curve $x = 2t, y = 3 + t^2$

[5 Marks]

c) Find the poles of $\frac{2z^2+5}{z^4+16}$ [4 Marks]

d) Expand $f(z) = \frac{3}{(z+2)(z-3)^2}$ in a Laurent Series at $z = 3$. [5 Marks]