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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2024/2025ACADEMIC YEAR
FIRST YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS AND COMPUTER SCIENCE)

Date: 18th April, 2024
Time: 8.30am –10.30am

KMA 103 - LINEAR ALGEBRA 1

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) For what value of k is the matrix

$$\begin{bmatrix} 4 & -3 & -1 \\ 2 & 4 & k \\ 3 & -5 & 4 \end{bmatrix}$$ is singular? (4 Marks)

b) Given the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x_1, x_2, x_3) = (x_1 - 2x_2, x_2 + 3x_3)$, find:

i) Kernel (T). (2 Marks)

ii) Nullity (T). (1 Mark)

iii) Rank (T). (1 Mark)

c) What condition must be satisfied by a, b and c so that the system of equations

$$x - 2y - 3z = a$$
$$2x + 6y - 11z = b$$
$$x - 2y + 7z = c$$ has a solution? (4 Marks)

d) Determine which of the following are subspaces of $\mathbb{R}^3$?

i) All vectors of the form $(a, 0, 0)$. (1 Mark)

ii) All vectors of the form $(a, 1, 1)$. (1 Mark)

iii) All vectors of the form $(a, b, c,)$, where $b = a + c$. (2 Mark)

e) Find the dimension of the row space of the matrix $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 5 \\ -1 & 2 & 2 \end{bmatrix}$.

(3 Marks)

f) Show that the vector $(-7,-6)$ is a linear combination of the vectors $(-2,3)$ and $(1,4)$.

(3 marks)

g) Solve the following linear system of equations using row reduction method.

$$x_1 + x_2 + x_3 = 3$$
$$3x_1 + 2x_2 + x_3 = 3$$
$$x_1 - 2x_2 - 5x_3 = 1$$

(5 marks)

h) Solve the following system of linear equations using Cramer's rule.

$$2x + 4y = -4$$
$$5x + 4y = 11$$

(3 Marks)

QUESTION TWO (20 MARKS)

a) Is $-1+x^2$ in the span of $p = 1 + x + x^3$ *and* $q = -x - x^2 - x^3$ in P_3?

(5 Marks)

b) Given the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}.$$ Evaluate;

i) the determinant of A (2 Marks)

ii) the inverse of A (2 Marks)

iii) hence or otherwise solve the system

$$2x + y + z = 2$$
$$2x \quad + 2z = 1$$
$$x + 2y + z = 0$$

(3 Marks)

c) Find the inverse of the following matrix by cofactor expansion method.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$$

(5 Marks)

d) Is $W = \{(a,b,3) : a,b \in \mathbb{R}\}$ *a subspace of* $\mathbb{R}^3$?

(3 Marks)

QUESTION THREE (20 MARKS)

a) Solve the following system of linear equations using Cramer's rule.

$$5x_1 - 2x_2 + x_3 = 1$$
$$3x_1 - 2x_2 = 3$$
$$x_1 + x_2 - x_3 = 0$$

(5 Marks)

b) Determine if b is a linear combination of the vectors

$$u_1 = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}, u_2 = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} 3 \\ 7 \\ 5 \end{bmatrix}, where\ b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

(5 Marks)

c) Compute the inverse of the following matrix using row reduction method

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & 2 & 1 \end{bmatrix}.$$

(5 Marks)

d) Determine the value(s) of h and k such that the system

$$x_1 + hx_2 = 1$$
$$2x_1 + 3x_2 = k$$

i) No solution. (3 marks)

ii) Many solutions.

(2 marks)

QUESTION FOUR (20 MARKS)

a) i) Define linear independence of vectors.

(1 Mark)

ii) Determine whether the set of vectors $\{(3,1,1), (2,-1,5), (4,0,-3)\}$ is linearly dependent in $\mathbb{R}^3$. (5 Marks)

b) Determine a basis and dimension of the solution space to

$$x_1 + x_2 - x_3 = 0$$
$$-2x_1 - x_2 + 2x_3 = 0$$
$$-x_1 + x_3 = 0$$

(5 Marks)

c) Use Gaussian elimination method to solve the following system of linear equations

$$x_1 + x_2 + x_3 = 9$$
$$2x_1 + 4x_2 - 3x_3 = 1$$
$$3x_1 + 6x_2 - 5x_3 = 0$$

(5 Marks)

d) Given the vectors $v_1 = [a, 1]$, $v_2 = [1, a]$, determine the values of a for which the vectors form a basis of $\mathbb{R}^2$. (4marks)

QUESTION FIVE(20 MARKS)

a) Let $f: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined as

$$f(x) = \begin{bmatrix} 1 & -1 & 2 \\ 0 & -1 & 1 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Find:

i) Basis for range of f. (3 Marks)

ii) Basis for kernel of f. (3 Marks)

iii) Rank (f) (3 Marks)

iv) nullity (f). (3 Marks)

b) Find the values of k for which the matrix $A = \begin{bmatrix} k-3 & 4 \\ k & k+2 \end{bmatrix}$ has no inverse.

(4 Marks)

c) Find the value of k for which the following matrix A is non-singular.

$$A = \begin{bmatrix} 1 & 2 & k \\ 3 & -1 & 1 \\ 5 & 3 & 5 \end{bmatrix}$$

(4 Marks)