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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2020/2021 ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS)

Date: 10th December, 2020
Time: 11.30am – 1.30pm

KMA 202 - VECTOR ANALYSIS

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) If $A = 3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}, B = 2\boldsymbol{i} + 5\boldsymbol{j} + \boldsymbol{k},$ and $C = \boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k},$ find $A \times (B \times C)$. (3 Marks)

b) Find the angles which the vector $A = 3\boldsymbol{i} - 6\boldsymbol{j} + 2\boldsymbol{k}$ makes with x-axis. (4 Marks)

c) Find the area of a triangle having vertices at P(1, 3, 2), Q(2, -1, 1) and R(-1, 2, 3). (4 Marks)

d) The acceleration of a particle at any time $t \geq 0$ is given by

$$a = 12\cos 2t\,\boldsymbol{i} - 8\sin 2t\,\boldsymbol{j} + 16t\,\boldsymbol{k}$$

If the velocity *v* and displacement *r* are zero at t=0. Find *v* and *r* at any time. (3 Marks)

e) If $\phi = 2xyz^2$, $F = xyi - zj + x^2k$ and C is the curve $x = t^3, y = 2t,\ z = t^2$ from t=0 to t=1, evaluate the line integral;

i) $\int_c \phi dr$ (3 Marks)

ii) $\int_c F \times dr$ (3 Marks)

f) Determine a unit vector perpendicular to the plane of $A = 2i - 6j - 3k$ and $B = 4i + 3j - k.$ (4 Marks)

g) Evaluate $\iiint_V (2x + y)dV,$ where V is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0, y = 0, y = 2$ and $z = 0.$ (6 Marks)

QUESTION TWO (20 MARKS)

a) A particle moves along a curve whose parametric equations are $x = 2\sin 3t\ , y = 2\ cos\ 3t,\ z = 8t,$ where t is the time.

i) Determine its velocity and acceleration at any time. (3 Marks)

ii) Find the magnitudes of the velocity and acceleration . (2 Marks)

b) If $\frac{d^2A}{dt^2} = 6t\ i - 24\ t^2 j + 4\sin t\ k$, find A given that $A = 2i + j$ and $\frac{dA}{dt} = -i - 3k$ at $t = 0$. (5 Marks)

c) If $A = 5t^2 + tj - t^3 k$ and $B = i - tj$ Find $\frac{d}{dt}(A \times B)$. (5 Marks)

d) If $\varphi(x, y, z) = xy^2 z$ and $A = xzi - xy^2 j + yz^2$k. find $\frac{\partial^3}{\partial x^2 \partial z}\ (\varphi A)$ at the point (2,-1,1). (5 Marks)

QUESTION THREE (20 MARKS)

a) If $\emptyset(x, y, z) = 3x^2 y - y^3 z^2$, find $\nabla\emptyset$ at the point (1,-2,-1). (5 Marks)

b) Find an equation for the tangent plane to the surface $xz^2 + x^2 y = z - 1$ at the point (1,-3,2). (5 Marks)

c) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9\ and\ z = x^2 + y^2 - 3$ at the point (2,-1,2). (5 Marks)

d) Determine the directional derivative of $\varphi = x^2 yz + 4xz^2$ at (1,-2,-1) in the direction $2i - j - 3k$ (5 Marks)

QUESTION FOUR (20 MARKS)

a) If $A = xz^4\boldsymbol{i} - 2x^3 yz\ \boldsymbol{j} + 2yz^2\boldsymbol{k}$, find $curl\ A$. (5 Marks)

b) If $F = (2x^2 - 3)\boldsymbol{i} - 2xy\ \boldsymbol{j} - 4x\ \boldsymbol{k}$, evaluate $\iiint_V \nabla \times F\ dV$, where V is the closed region bounded by the planes $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$. (6 Marks)

c) If $A = (3x^2 + 6y)\boldsymbol{i} - 14yz\ \boldsymbol{j} + 20\ xz^2\boldsymbol{k}$, evaluate $\int_c\ A.dr$ along the straight lines from (0,0,0) to (1, 1, 1). (5 Marks)

d) If $F = 3xy\ \boldsymbol{i} - y^2\boldsymbol{j}$, evaluate $\int_c\ F.dr$ where c is the curve in the xy-plane, $y = 2x^2$ from$(0,0)\ \ to\ (1,2)$. (4 Marks)

QUESTION FIVE (20 MARKS)

a) State the Green's theorem and hence verify it in the plane for$\oint_c\ (3x^2 + 8y^2)dx + (4y - 6xy)dy$,where C is the boundary of the region defined by $y = \sqrt{x}$ and $y = x^2$. (7 Marks)

b) Find the total work done in moving a particle in a force field given by $F = 3xy\ i - 5zj + 10x\ k$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 0\ to\ t = 1$. (5 Marks)

c) Given $\emptyset = 2x^3 y^2 z^4$. Find $\nabla \cdot \nabla\emptyset(or\ div\ grad\ \emptyset)\ at\ (1,-2,1)$ (5 Marks)

d) Find a unit vector parallel to the resultant of the vectors $r_1 = 2i + 4j - 5k,\quad r_2 = i + 2j + 3k$ (3 Marks)