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**KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY**  
UNIVERSITY EXAMINATION, 2020/2021 ACADEMIC YEAR  
FIRST YEAR, FIRST SEMESTER EXAMINATION  
FOR THE DEGREE OF BACHELOR OF SCIENCE  
(MATHEMATICS)

Date: 17<sup>th</sup> December, 2020  
Time: 11.30am – 1.30pm

**KMA 104 - CALCULUS 1**

**INSTRUCTIONS TO CANDIDATES**

**ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS**

**QUESTION ONE (30 MARKS)**

- a) Explain each of the following terms as used in calculus
- i) Differentiation (1 Mark)
  - ii) Function (1 Mark)
  - iii) Limit point of a function (1 Mark)
- b) Find the exact value of  $\lim_{x \rightarrow \infty} \sqrt{x^3 - 4} - \sqrt{x^3 + x}$  (4 Marks)
- c) Find  $\frac{dy}{dx}$  if  $y = \sec x$  (4 Marks)
- d) Evaluate  $\lim_{x \rightarrow 0} \left( \frac{\sqrt{1+x} - \sqrt{1-x}}{x} \right)$  (3 Marks)
- e) Evaluate the derivative of  $x + y^2 = e^{xy^3}$  (4 Marks)
- f) From the definition of derivatives, find the derivative of  $f(x) = 3 - 2x - x^2$  (3 Marks)
- g) Let  $x = t^3$  and  $y = t^2 - t$ . Obtain the first derivative of  $y$  with respect to  $x$  (3 Marks)
- h) Find the derivatives of the following
- i)  $y = \ln(x^2 + 1)$  (2 Marks)
  - ii)  $y = \frac{1 - \cos x}{1 + \cos x}$  (2 Marks)
  - iii)  $y = x^3(2x - x^4)$  (2 Marks)

**QUESTION TWO (20 MARKS)**

- a) Sand is falling in a conical pipe at a rate of  $100m^3 / \text{min}$ . Find the rate of change of the height when the height is  $10m$  assuming that the coasance of the sand is such that the sand is equal to the radius. (12 Marks)
- b) Evaluate  $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 4}$  (4 Marks)
- c) Find the derivative of the following function  $x^2 + y^2 = 1$  (4 Marks)

**QUESTION THREE (20 MARKS)**

- a) A drug is injected into the bloodstream of a patient through the right arm. The concentration of the drug in the bloodstream of the left arm  $t$  hours after the injection is approximated by  $c(t) = \frac{0.14t}{t^2 + 1}$   $0 \leq t \leq 24$ . Determine  $c(3)$  and interpret your results (3 Marks)
- b) Find the derivative of each of the following equations
- i)  $y = \frac{3 - 2x^2}{3 + 2x}$  (5 Marks)
- ii)  $x = y\sqrt{1 - y^3}$  (5 Marks)
- c) The position function of a particle is given by  $s(t) = 3t^2 - t^3, t \geq 0$
- i) When does the particle reach a velocity of  $0m/s$ ? Explain the significance of this value of  $t$  (4 Marks)
- ii) When does the particle have acceleration  $0m/s^2$ ? (3 Marks)

**QUESTION FOUR (20 MARKS)**

- a) Using the function  $f(x) = 25 - 5x^2$  find
- i) Its derivative from the first principle (7 Marks)
- ii) The rate of change of  $f(x)$  at  $x = 1$  (1 Mark)
- b) Differentiate the following functions with respect to  $x$
- i)  $y = \left(\frac{x}{1+x}\right)^5$  (4 Marks)
- ii)  $y = \cos^2 3x$  (4 Marks)
- c) Find the equation of the normal to the curve  $y = x^2 + 3x$  at the point where  $x = 1$  (4 Marks)

**QUESTION FIVE (20 MARKS)**

- a) Find the equation of the tangent line to the graph of  $f(x) = \sqrt{x^2 + 3}$  at the point  $(-1, 2)$  (6 Marks)
- b) Find all critical points of  $f(x) = x^4 - 8x^2$ . (5 Marks)
- c) Evaluate  $\lim_{x \rightarrow -3} \frac{x^2 - 9}{x + 3}$  (4 Marks)
- d) Using the definition of a derivative, find the derivative of  $f(x) = x^2$ . (5 Marks)