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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
FOURTH YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN
MATHEMATICS
KMA 424: OPERATION RESEARCH III

Date:

Time:

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) Discuss the concept of cost-benefit analysis. (3 Marks)
b) With reference to game theory, describe the properties of a competitive game. (4 Marks)
c) Solve the following payoff matrix using the Maximin-Minimax method;

		Player B		
		I	II	III
Player A	I	-3	-2	6
	II	2	0	4
	III	5	-2	-4

Determine the;

- i) value of the game (3 Marks)
ii) optimal strategies for A and B (2 Marks)
d) Write down the general formulation of multi-objective optimization (2 Marks)
e) Discuss five assumptions underlying consumer's utility through maximization of their objective. (5 Marks)
f) Using the principle of dominance, solve the following payoff matrix

		Q		
		I	II	III
P	I	5	20	-10
	II	10	6	3
	III	20	15	18

Calculate the;

- i) Value of the game (4 Marks)
ii) Probabilities for the strategies of P and of Q (4 Marks)
g) Given the utility function $U=f(x, y)$, show that the marginal rate of substitution is a ratio of the marginal utilities of commodities involved. (3 Marks)

QUESTION TWO (20 MARKS)

- a) Differentiate between pure and mixed integer programming. (2 Marks)
b) Describe the steps involved in Cutting-Plane Algorithm in solving an integer programming problem. (7 Marks)
c) Consider the following IP problem,

$$\max z = 80x_1 + 45x_2$$

subject to;

$$x_1 + x_2 \leq 7$$
$$12x_1 + 5x_2 \leq 60$$

$$x_1, x_2 \geq 0 \wedge \text{integer}$$

Using graphical representation, find the set of solutions for this problem.

(5 Marks)

d) Discuss the three general decision environment categories.

(6 Marks)

QUESTION THREE (20 MARKS)

a) Discuss the Cost – Benefit Analysis and the steps for carrying out the analysis (6 Marks)

b) The table below details the utility derived from a bundle of commodities chosen daily by a consumer

Day	Total Utility	Marginal Utility
1	0	0
2	10	10
3	22	
4	37	
5		18
6	63	
7	60	

i) Complete the table

(2 Marks)

ii) Explain the law of diminishing marginal utility citing where it appears in the table above

(3

Marks)

c) Solve the following game using graphical method

		B			
A		B1	B2	B3	B4
	A1	2	5	8	10
	A2	9	4	2	0

What is the;

i) Value of the game

(5 Marks)

ii) Expected payoffs for A and B

(4 Marks)

QUESTION FOUR (20 MARKS)

a) Discuss the difference between cardinal and ordinal theories of utility (4 Marks)

b) A decision is being made concerning which size facility should be constructed. The following table gives the present value (in millions) for each alternative under each state of nature.

Alternatives	Possible Future Demand		
	Low	Moderate	High
Small Facility	12	9	11
Medium Facility	8	11	13
Large Facility	5	3	18

Using Minimax Regret criterion for decision making,

i) Construct a regret /opportunity loss table

(4 Marks)

ii) Identify the worst regret for each alternative hence determine the size of the facility to be constructed.

(4 Marks)

c) Suppose you are running a factory, producing some sort of widget that requires steel as a raw material. Your costs are predominantly human labor, KES 20 per hour for your workers, and the steel itself, which runs for KES 170 per ton. Suppose your revenue R is loosely modeled by the following equation:

$$R(h, s) = 200 h^{\frac{2}{3}} s^{\frac{1}{3}}$$

where h represents hours of labour s represents tons of steel

If your budget is KES 20000,

i) Write down the constraint and the Lagrangian function

(3 Mark)

ii) What is the maximum possible revenue?

(5 Marks)

QUESTION FIVE (20 MARKS)

a) Discuss how one realizes Nash Equilibrium with reference to the Prisoners' Dilemma.

(5

Marks)

b) Giving an example for each, distinguish between consumer surplus and producer surplus as used in economics.

(4 Marks)

c) Given the general optimization problem

$$\begin{aligned} & \min_{x \in R^n} f(x) \\ & \text{subject to} \\ & h_i(x) \leq 0, i=1,2,\dots,m \\ & l_j(x)=0, j=1,\dots,r \end{aligned}$$

Write down;

i) The Karush-Kuhn-Tucker conditions.

(4 Marks)

ii) The necessary and sufficient conditions for primal and dual solutions

(4 Marks)

d) Explain Markov Chain draw the Markov process for the given transition matrix

$$\begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$$

(3 Marks)