KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
FOURTH YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS)

Date: 5th December, 2023
Time: 8.30am –10.30am

KMA 409 - DESIGN AND ANALYSIS OF EXPERIMENTS

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a)   Briefly explain the importance of replication and randomization as principles in design and analysis of experiments.

(5 Marks)

b)   In a completely randomized design, seven experimental units (number of observations for each category) were used for each of the six levels (number of categories). Below is an incomplete ANOVA table for the experiment.

Source of variation	Sum of squares	Degrees of freedom	Mean square	F
Treatments	300			
Error				
Total	650			

Find the;
i) Degrees of freedom for error.   (2 Marks)
ii) Mean squares due to error (MSE).   (2 Marks)
iii) Degrees of freedom of treatment.   (2 Marks)
iv) Critical value for the test for α = .05.   (2 Marks)

c)   Explain why a B.I.B design is preferred in analysis of variance compared to an R.B.D

(3 Marks)

d)   When are two Latin squares said to be orthogonal?

(3 Marks)

e)   Briefly described what is meant by a 2^3 factorial design using relevant practical example.

(4 Marks)

f) A study of depression and exercise was conducted. Four groups were used. A depression rating was given to members in each group. Small random samples from each groups provided the following data:

Cycle Group:	63	58	61
Sedentary Group:	71	64	68
Runners:	49	52	47
Walkers:	45	43	49

i) State the null and alternative hypothesis to be tested for this investigation.

(2 Marks)

ii) Use ANOVA to test if the mean depression ratings for the four groups are different.
Use $\alpha = 0.05$. (5 Marks)

QUESTION TWO (20 MARKS)

a) Give the model used in the analysis of randomized complete block design clearly defining each symbol. (5 Marks)

b) What are the advantages of randomized complete block design over a completely randomised design? (5 Marks)

c) An engineer is studying the mileage performance characteristics of 5 types of Gasoline additives. In the road tests he wishes to use cars as blocks, however because of time constraints, he must use an incomplete block design. He runs the balanced design with the 5 blocks that follow. Analyse the data from this experiment (use $\alpha = 0.05$) and draw conclusions.

Additive	Blocks (cars)				
	1	2	3	4	5
1	-	17	14	13	12
2	14	14	-	13	10
3	12	-	13	12	9
4	13	11	11	12	-
5	11	12	10	-	8

(10 Marks)

QUESTION THREE (20 MARKS)

a) Consider a Latin square design with mode $y_{ijk} = \mu + \alpha_i + \beta_j + T_k + \epsilon_{ij}$ (usual meaning of symbols) $i = 1,2, \dots, s; \quad j = 1,2, \dots, s \quad k = 1,2, \dots, s$. Suppose that the observation in k^{th} treament and i^{th} row belonging and j^{th} column is missing. Obtain an estimator of the missing observation. (10 Marks)

b) Consider the R.B.D shown below, with observation belonging to the 2nd treatment in the 2nd block missing. Evaluate the missing observation.

		Treatments			
		1	2	3	4
Blocks	1	9.3	9.4	9.6	10.0
	2	9.4	-	9.3	9.9
	3	9.2	9.4	9.5	9.7
	4	9.7	9.6	10.0	10.2

(5 Marks)

c)   Consider two mutually orthogonal Latin squares

L₁		
A	B	C
B	C	A
C	A	B

L₂		
∝	β	γ
γ	∝	β
β	γ	∝

Using this Latin squares, obtain a B.I.B design with parameters
$b = 12, v = 9, k = 3, r = 4, \lambda = 1$.

(5 Marks)

QUESTION FOUR (20 MARKS)

a)   This experiment below investigates a procedure to create artificial arteries using a resin. The resin is pressed or extruded through an aperture that forms the resin into a tube. To conduct this experiment, all 4 pressures were assigned at random to each of the 6 batches of resin. Each batch of resin is called a "block", since a batch is a more homogenous set of experimental units on which to test the extrusion pressures. Below is a table which provides percentages of those products that met the specifications.

Extrusion pressure (PSI)	Batch of Resin				
	1	2	3	4	5
8500	90.3	89.2	98.2	93.9	87.4
8700	92.5	89.5	90.6	94.7	87
8900	85.5	90.8	89.6	86.2	88
9100	82.5	89.5	85.6	87.4	78.9

The above experimental data was analysed using a statistical software and the following output was obtained

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Block	192.25	5	38.45	5.25	0.006
Treatment	178.17	3	59.39	8.11	0.0019
Error	109.89	15	7.33		
Total	480.31	23			

Std. Dev. 2.71   R-Squared   0.6185
Mean   89.80 Adj R-Squared   0.5422
C.V.   3.01   Pred R-Squared 0.0234
PRESS   281.31 Adeq. Precision 9.759

Discuss the output under the following subheadings
i) A customized model of analysis.   (2 Marks)
ii) Hypothesis to be tested making appropriate conclusions based on the critical values.
                                                                                       (6 Marks)

iii) Adequacy of the model.
                                                                                       (2 Marks)

b) A farmer wishes to test the effects of 4 fertilizers (A, B, C and D) on the yield of wheat. In order to eliminate sources of error due to variability in soil fertility, he uses the fertilizers in a Latin square arrangement. Perform analysis of variance to determine whether there is a difference between fertilizers at 5% level of significance.

A18	C21	D25	B11
D22	B12	A15	C19
B15	A20	C23	D24
C22	D21	B10	A17

(10 Marks)

QUESTION FIVE (20 MARKS)

a) Briefly explain the following terms used in factorial designs.

i)   Design matrix.   (2 Marks)

ii)   X-matrix.   (2 Marks)

iii)   Simple effects of a factor.   (2 Marks)

b) Consider an investigation into the effects of the concentration of reactants in the presence of catalysts on time of a chemical process. Let the reactants concentration be at levels 15% and 25% while the catalysts be at high and low levels.

Treatments	I	II	III
1	28	25	27
a	36	32	32
b	18	19	23
ab	31	30	29

i) Estimate the main effects and the interaction effects.   (5 Marks)

ii) Find the sum of squares of the main and the interaction effects.   (5 Marks)

iii) Prepare the analysis of variance table for the experiment.   (4 Marks)