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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
FIRST YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS AND COMPUTER SCIENCE)

Date: 5th December, 2023
Time: 11.30am –1.30pm

KMA 106 -PROBABILITY AND STATISTICS 1

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) A random variable X has a probability distribution given by

$$f(x) = \begin{cases} cx(1-x), & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find;

- i) Value of the constant c. (3 marks)
- ii) $P\left(\frac{1}{2} \leq X \leq \frac{2}{3}\right)$ (3 marks)
- b) Suppose that a random variable X has a probability mass function given by
- $$f(x) = \begin{cases} \frac{2x+3}{35}, & x = 0, 1, 2, 3, 4 \\ 0, & \text{elsewhere} \end{cases}$$
- i) Obtain the moment generating function of X. (3 marks)
- ii) Use the moment generating function to determine;
- i) Expected value of X. (2 marks)
- ii) Variance of X. (3 marks)
- c) Suppose, according to the latest police reports, 80% of all petty crimes are unresolved. In your town, at least three of such petty crimes are committed. The three crimes are all independent of each other. What is the probability that at most one of the three crimes will be resolved? (3 marks)
- d) Customers arrive at a checkout counter according to a Poisson distribution at an average of 7 per hour. During a given hour, what are the probabilities that at least 2 customers arrive? (3 marks)
- e) The number of miles that a particular car can run before its battery wears out is exponentially distributed with an average of 10,000 miles. The owner of the car needs to take a 5000-mile trip. What is the probability that he will be able to complete the trip without having to replace the car battery? (3 marks)

- f) Most graduate schools of business require applicants for admission to take the Graduate Management Admission Council's GMAT examination. Scores on the GMAT are roughly normally distributed with a mean of 527 and a standard deviation of 112. What is the probability of an individual scoring above 500 on the GMAT? (3 marks)
- g) Pepsi's dispensing machine is designed to fill bottles with exactly 2 litres of their product. To test if the machine is performing according to specification, a sample of 100 "2-litre" bottles is collected. The average quantity contained in the sample bottles is $\bar{X} = 1.985$ litres. The population standard deviation of the fill is known to be 0.05. Test whether the machine is in control, at the 5% level of significance. (4 marks)

QUESTION TWO (20 MARKS)

- a) A random variable X has a Poisson distribution given by

$$P(X = x) = \begin{cases} \frac{e^{-\lambda} \lambda^x}{x!}, & x = 0, 1, 2, \dots \\ 0, & \text{Otherwise} \end{cases}$$

Without using moment generating function, show that

- i) Mean of X is λ . (5 marks)
- ii) Variance of X is also λ . (5 marks)
- b) Suppose a given website receives an average of 20 visitors per hour. What is the probability that the number of visitors received by the website is;
- i) At least 4 in an hour? (3 marks)
- ii) Exactly 15 in three hours? (3 marks)
- c) A deck of cards contains 20 cards; 6 red cards and 14 black cards. 5 cards are drawn randomly *without replacement*. What is the probability that exactly 4 red cards are drawn? (4 marks)

QUESTION THREE (20 MARKS)

- a) The final exam scores in a statistics class were normally distributed with a mean of 63 and a standard deviation of 5.
- i) Find the probability that a randomly selected student scored more than 65 on the exam. (3 marks)
- ii) Find the probability that a randomly selected student score between 60 and 70. (3 marks)
- iii) If 90% of students scored below grade k, what is the maximum value of k? (3 marks)
- b) Suppose you play a game that you can only either win or lose. The probability that you win any game is 55%, and the probability that you lose is 45%, what is the probability that you win;
- i) 15 times if you play the game 20 times? (3 marks)
- ii) between 16 and 18 (inclusive) of all 20 games? (3 marks)
- c) The average number of donuts a nine-year old child eats per month is uniformly distributed from 1 to 4 donuts, inclusive.
- i) Determine the probability that a randomly selected nine-year old child eats an average of between 0 and 2 donuts. (2 marks)
- ii) What is the expected number and the standard deviation of donuts consumed in a day? (3 marks)

QUESTION FOUR (20 MARKS)

- a) Highlight the steps followed in hypothesis testing. (4marks)
- b) A simple random sample of 10 people from a certain population has a mean age of 27. Can we conclude that the mean age of the population is less than 30? The variance is known to be 20. Let $\alpha = .05$. (4 marks)
- c) The following figures give the end of year profits of ten randomly selected Chemists in Nairobi County.

Profit (Million Shillings)	21.8	24.8	27.3	29.3	30.8	31.8	32.8	32.5	32.1	31.3
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On the basis of this data, test whether the average profit is 30M KSH at 1% level of significance. (5 marks)

- d) Big Foods Grocery has two grocery stores located in Johnston City. One store is located on First Street and the other on Main Street and each is run by a different manager. Each manager claims that her store's layout maximizes the amounts customers will purchase on impulse. Both managers surveyed a sample of their customers and asked them how much more they spent than they had planned to, in other words, how much did they spend on impulse? The following table shows the sample data collected from the two stores

First Street	15.78	17.73	10.61	15.79	14.22	13.82				
Main Street	15.19	18.22	15.38	15.96	21.92	12.87	18.4	18.57	17.79	10.83

Upper-level management at Big Foods Grocery wants to know if there is a difference in the mean amounts purchased on impulse at the two stores. Assuming that the two stores have a common but unknown variance, test at 5% whether there is a difference in the mean amounts purchased on impulse at the two stores. (7 marks)

QUESTION FIVE (20 MARKS)

- a) Consider a random a random variable X with a probability density function given by
$$f(x) = \begin{cases} 4 e^{-4x}, & x > 0 \\ 0, & \text{Otherwise} \end{cases}$$
 - I) Find the moment generating function of X. (4 marks)
 - II) Use the moment generating function to find
 - i) Mean of X. (3 marks)
 - ii) Variance of X. (3 marks)
- b) The time that a customer takes to be served at a service counter is exponentially distributed with an average of 3 minutes. Of 100 customers, how many do you expect to be served for more than 5 minutes? (3 marks)
- c) A random variable X has a probability distribution given by

x	2	3	4	5	6
P(X=x)	0.01	0.25	0.4	0.3	0.04

Find

- i) Determine the cumulative distribution (c.d.f) of X and sketch it. (3 marks)
- ii) Use the c.d.f in (I) to find
 - i) Median of X. (2 marks)
 - ii) 87th percentile of X. (1 mark)
- iii) What is the mode of X? (1 mark)