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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2020/2021 ACADEMIC YEAR
THIRD YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS)

Date: 14th December, 2020
Time: 11.30am – 1.30pm

KMA 2304 - NUMBER THEORY

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) Explain with examples the meaning of the following terms as used in number theory
- i) Natural numbers (1 Mark)
 - ii) Rational numbers (1 Mark)
 - iii) Composite numbers (1 Mark)
 - iv) Integers (1 Mark)
- b) Show that if $\frac{d}{a}$ and $\frac{d}{b}$, then $\frac{d}{ra \pm sb}$ (4 Marks)
- c) With an example show that every composite integer n has a prime p such that $1 < p < \sqrt{n}$ (4 Marks)
- d) For positive integers 654 and 381, show that $(654, 381) = 3$ (4 Marks)
- e) Prove that if $\frac{n}{ab}$ and n and a are coprime, then $\frac{n}{b}$ (5 Marks)
- f) State the Wilson's theorem (2 Marks)
- g) Solve for $x^2 + y^2 \equiv 0 \pmod{3}$ (3 Marks)

QUESTION TWO (20 MARKS)

- a) Find all the right-angled triangles with integer sides and a perimeter of 240 (7 Marks)
- b) Discuss the Pell's equation hence solve $x^2 - 2y^2 = 1$ (8 Marks)
- c) Find the GCD of the two numbers (37129,14659) using Euclidean algorithm. (5 Marks)

QUESTION THREE (20 MARKS)

- a) If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, show that $a \pm c \equiv b \pm d \pmod{m}$ (4 Marks)
- b) Solve $x \equiv 4 \pmod{21}$ and $x \equiv 13 \pmod{30}$ simultaneously (6 Marks)
- c) Find the solutions of the linear Diophantine equation $109x + 87y = 50001$ (6 Marks)
- d) By considering $f(x, y) \pmod{4}$ show that $f(x, y) = y^2 - x^2 - 2 = 0$ has no solution. (4 Marks)

QUESTION FOUR (20 MARKS)

- a) Define pseudo-prime (2 Marks)
- b) State the Fermat's theorem hence show that if p is prime, then $2^p \equiv 2 \pmod{p}$ (6 Marks)
- c) Solve $3^x \equiv 2 \pmod{11}$ and $3^x \equiv 5 \pmod{11}$ (6 Marks)
- d) Show that we cannot have 3 consecutive odd numbers other than 3,5,7 such that they are all prime. (6 Marks)

QUESTION FIVE (20 MARKS)

- a) State Helly's theorem (2 Marks)
- b) Solve $3x - 5y + 7z = 12$, $5x + 9y - 11z = 40$ simultaneously (8 Marks)
- c) Show that $\sqrt{568}$ is a prime number (5 Marks)
- d) If a/b and c/d is it true that $a + c/b + d$. (5 Marks)