Kasarani Campus
Off Thika Road
Tel. 2042692 / 3
P. O. Box 49274, 00100
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2020/2021 ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS)

Date: 14th December, 2020
Time: 11.30am – 1.30pm

KMA 205 - BASIC NUMBER THEORY

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) Explain with examples the meaning of the following terms as used in number theory
   i) Natural numbers (1 Mark)
   ii) Rational numbers (1 Mark)
   iii) Composite numbers (1 Mark)
   iv) Integers (1 Mark)

b) Show that if $\frac{d}{a}$ and $\frac{d}{b}$, then $\frac{d}{ra \pm sb}$ (4 Marks)

c) With an example show that every composite integer n has a prime p such that $1 < p < \sqrt{n}$ (4 Marks)

d) For positive integers 654 and 381, show that $(654,381) = 3$ (4 Marks)

e) Prove that if $\frac{n}{ab}$ and n and a are coprime, then $\frac{n}{b}$ (5 Marks)

f) State the Wilson's theorem (2 Marks)

g) Solve for $x^2 + y^2 \cong 0 (\bmod 3)$ (3 Marks)

QUESTION TWO (20 MARKS)

a) Find all the right-angled triangles with integer sides and a perimeter of 240 (7 Marks)

b) Discuss the pell's equation hence solve $x^2 - 2y^2 = 1$ (8 Marks)

c) Find the GCD of the two numbers (37129,14659) using Euclidean algorithm. (5 Marks)

QUESTION THREE (20 MARKS)

a) If $a \cong b(\bmod m)$ and $c \cong d(\bmod m)$, show that $a \pm c \cong b \pm d(\bmod m)$ (4 Marks)

b) Solve $x \cong 4(\bmod 21)$ and $x \cong 13(\bmod 30)$ simultaneously (6 Marks)

c) Find the solutions of the linear Diophantine equation $109x + 87y = 50001$ (6 Marks)

d) By considering $f(x, y)\ mod\ 4$ show that $f(x, y) = y^2 - x^2 - 2 = 0$ has no solution. (4 Marks)

QUESTION FOUR (20 MARKS)

a) Define pseudo-prime (2 Marks)

b) State the Fermat's theorem hence show that if p is prime, then $2^p \cong 2(\bmod p)$ (6 Marks)

c) Solve $3^x \cong 2(\bmod 11)$ and $3^x \cong 5(\bmod 11)$ (6 Marks)

d) Show that we cannot have 3 consecutive odd numbers other than $3,5,7$ such that they are all prime. (6 Marks)

QUESTION FIVE (20 MARKS)

a) State Helly's theorem (2 Marks)

b) Solve $3x - 5y + 7z = 12, \quad 5x + 9y - 11z = 40$ simultaneously (8 Marks)

c) Show that $\sqrt{568}$ is a prime number (5 Marks)

d) If a / b and c / d is it true that $a + c/b + d$. (5 Marks)