



Kasarani Campus
Off Thika Road
Tel. 2042692 / 3
P. O. Box 49274, 00100
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2020/2021 ACADEMIC YEAR
FIRST YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS)

Date: 17th December, 2020
Time: 2.30pm – 4.30pm

KMA 106 - PROBABILITY AND STATISTICS 1

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) Define each of the following terms
- i) Random variable (1 Mark)
 - ii) Expectation (1 Mark)
 - iii) Moment generating function. (1 Mark)
- b) A continuous random variable X has a probability density function given by
- $$f(x) = \begin{cases} 5kx(1-x) & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$
- Determine the value of the constant k and the mode of this distribution (5 Marks)
- c) The probability density function of random variable X is defined as
- $$f(x) = \begin{cases} 3x^2, & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$
- i) Show that $f(x)$ is indeed a probability density function (2 Marks)
 - ii) Find the mean and variance of X (4 Marks)
- d) Let Y have the probability density function $f(y) = \begin{cases} \frac{4}{8}y^2(4-y), & 0 \leq y < 4 \\ 0, & \text{elsewhere} \end{cases}$.
- Compute the mode of the above distribution (3 Marks)
- e) The top 5% of applicants (as measured by GRE scores) will receive scholarships. If $GRE \sim N(500, 100^2)$, how high does your GRE score have to be to qualify for a scholarship? (5 Marks)

- f) A balloon manufacturer claims that 95% of his balloons will not burst when blown up. If you have 20 of these balloons to blow up for a birthday party.
- i) What is the probability that none of them burst when blown up? (2 Marks)
 - ii) Find the probability that exactly 2 balloons burst. (2 Marks)
- g) A fertilizer mixing machine is set to give 12 kg of nitrate for every 100kg bag of fertilizer. Ten 100kg bags are examined. The percentages of nitrate are as follows: 11, 14, 13, 12, 13, 12, 13, 14, 11, 12. Is there reason to believe that the machine is defective at 5% level of significance? (4 Marks)

QUESTION TWO (20 MARKS)

- a) The probability density function for a continuous random variable X is

$$f(x) = \begin{cases} a + bx^2, & 0 \leq x \leq 1 \\ 0, & elsewhere \end{cases}$$

Find

- i) a, b if $E[X] = \frac{3}{5}$ (7 Marks)
 - ii) Variance of X (4 Marks)
- b) Consider the probability density function of X defined as

$$f(x) = \begin{cases} \frac{3}{2}e^{-6x}, & x > 0 \\ -\frac{3}{2}x, & -1 < x \leq 0 \\ 0, & otherwise \end{cases}$$

Find the cumulative distribution function of X . Hence, determine the $P(-0.5 \leq X \leq 0.5)$ (9 Marks)

QUESTION THREE (20 MARKS)

- a) Family income is believed to be normally distributed with a mean of \$25000 and a standard deviation on \$10000. If the poverty level is \$10,000;
- i) What percentage of the population lives in poverty? (4 Marks)
 - ii) A new tax law is expected to benefit “middle income” families, those with incomes between \$20,000 and \$30,000. What percentage of the population will benefit from the law? (4 Marks)
- b) The moment generating function of random variable X is given by $M_x(t) = (1 - 2\beta t)^{-\alpha}$. Obtain the mean and variance of X (6 Marks)
- c) A random variable X has the probability distribution function given by $f(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & elsewhere \end{cases}$.
- Find:
- i) Lower quartile (3 Marks)
 - ii) Median (3 Marks)

QUESTION FOUR (20 MARKS)

- a) Bits are sent over a communications channel in packets of 12. If the probability of a bit being corrupted over this channel is 0.1 and such errors are independent.
- i) What is the probability that no more than 2 bits in a packet are corrupted? (3 Marks)
 - ii) If 6 packets are sent over the channel, what is the probability that at least one packet will contain 3 or more corrupted bits? (3 Marks)
- b) A random variable X is a beta distribution with parameters $\alpha = 3$ and $\beta = 5$ state the mean of this distribution. (3 Marks)
- c) State any three properties of a poisson distribution (3 Marks)
- d) The university policy department must write, on average, five tickets per day to keep department revenues at budgeted levels. Suppose the number of tickets written per day follows a Poisson distribution with a mean of 8.8 tickets per day. Find the probability that
- i) Less than 2 tickets are written on a randomly selected day from this distribution. (3 Marks)
 - ii) Exactly 9 tickets are written in two days (2 Marks)
 - iii) At most four tickets are written in half a day (3 Marks)

QUESTION FIVE (20 MARKS)

- a) Distinguish between the following terms:
- i) Null hypothesis and alternative hypothesis (2 Marks)
 - ii) P-Value and level of significance (2 Marks)
- b) Outline the steps followed when testing hypothesis (3 Marks)
- c) A random sample of 121 checking accounts at a bank showed an average daily balance of KShs. 280. The standard deviation is known to be KShs. 60.
- i) Construct a 95% confidence interval for the average daily balance at the bank. (4 Marks)
 - ii) With 95% confidence, how many *more* checking accounts need to be included in the sample to provide a confidence interval with length 10? (3 Marks)
 - iii) Test the hypothesis $H_0 : \mu = 250$ against $H_1 : \mu \neq 250$ at $\alpha = 0.01$ (6 Marks)