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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
THIRD YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

KMA 302: COMPLEX ANALYSIS I

Date: 10th August, 2023
Time: 11.30am – 1.30pm

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Evaluate $\frac{1+i}{2-3i}+\frac{4-7i}{3+2i}$ (3 Marks)

b) Solve the equation $z^2-4z+53=0$.
Express the roots in the form $a+ib$ where $a,b\in R$. Verify that the sum of the roots is 4 and the product is 61. (3 Marks)

c) Find all values of ¿ hence state the principle value. (3 Marks)

d) Evaluate $\frac{\lim_{z\to 0} 1-\cos z}{\sin z^2}$. (3 Marks)

e) Show that $a^z=e^{z\ln a}$. (3 Marks)

f) Use Residue theorem to evaluate $\int_{-\infty}^{\infty}\frac{dz}{1+z^2}$ (4 Marks)

g) Show that $f(z)=e^{iz}$ is analytic. (3 Marks)

h) Evaluate $\int_c \bar{z}\,dz$ from $z=0$ $z=4+2i$ along the curve C given by $z=t^2+it$ (4 Marks)

i) Find the Laurent series of the function $\frac{e^{2z}}{(z-1)^3}$ about the singularity ¿1 .

Name the singularity and give the region of convergence. (4 Marks)

QUESTION TWO (20 MARKS)

a) Test the continuity of the function;

$$f(z)=\begin{cases}\frac{z^2+4}{z-2i};z\neq 2i\\ 3+4i;z=2i\end{cases}$$

i) Prove that $\lim_{z\to i} f(z)$ exists and determine it's value. (3 Marks)

ii) Is $f(z)$ continuous at $z=2i$? Explain. (2 Marks)

iii) Is $f(z)$ continuous at points $z\neq 2i$? Explain. (2 Marks)

b) Use Cauchy's Integral Formula to evaluate $\oint_c \frac{e^{2z}}{(z+1)^4}dz$ where C is the circle $|z|=3$ (6 Marks)

c) Simplify $\frac{(\cos 3\theta+i\sin 3\theta)(\cos 5\theta-i\sin 5\theta)}{(\cos 2\theta-i\sin 2\theta)(\cos 7\theta+i\sin 7\theta)}$ (4 Marks)

d) Write down Z given that $|Z|=2$ and $arg\,Z=150^\circ$ (3 Marks)

QUESTION THREE (20 MARKS)

a) If $w=f(z)=\frac{1+z}{1-z}$,

i) Find $\frac{dw}{dz}$ (2 Marks)

ii) Determine where $f(z)$ is non-analytic. (2 Marks)

b) Prove that a necessary condition for $f(z)=u(x,y)+iv(x,y)$ to be analytic in the region D is that the Cauchy Riemann equations $\frac{\partial u}{\partial x}=\frac{\frac{\partial v}{\partial x}\wedge\partial u}{\partial y}=\frac{-\partial v}{\partial x}$.

Hence show that $f(z)=2z^2+5$ is analytic. (9 Marks)

c) Evaluate using Residue Theorem;

$$\int_c \frac{e^{iz}}{z^2(z-2)(z+5i)}dz, C:|z|=1$$ (7 Marks)

QUESTION FOUR (20MARKS)

a) Show that if a function $f(z)$ is analytic inside and on a simple closed contour C, then $\int_c f(z)dz=0$ (6 Marks)

b) Evaluate the following integral $\int_c \frac{1}{z(z-1)(2z-1)}dz, C:|z|=2$ using

i) Cauchy's theorem (5 Marks)

ii) Cauchy's integral formula (4 Marks)

c) Show that $|\sin z|^2=\sin^2 x+\sin h^2 y$ (5 Marks)

QUESTION FIVE (20 MARKS)

a) Given $u(x,y)=x^3-3xy^2+3x^2-3y^2+1$ is harmonic, find $v(x,y)$ such that $f(z)$ is analytic. Express $f(z)$ in terms of z only. (6 Marks)

b) Expand $f(z)=\frac{1}{(z+2)(z-1)}$, in a Laurent's series valid for $1<|z-2|<4$. (7 Marks)

c) Use Residue theorem to evaluate the integral $\int_{o}^{2\pi} \frac{1}{5-3\sin\theta}\,d\theta$ (7 Marks)