a) **The two causes of variations in the variable being monitored in the production line**

i) **Assignable variation** refers to the variation that can be attributed to specific causes or factors that are under the control of an organization or individual. It is also known as special cause variation. These causes can be identified, monitored, and controlled to improve the performance or quality of a process or product. Examples include malfunctioning machine, inconsistent raw material quality, or operator errors

ii) **Common variation,** also known as random or natural variation, refers to the inherent variability present in a process or system. It is caused by a combination of numerous factors that are not easily identifiable or controllable. Common variation is considered to be stable over time and can be represented by a statistical distribution. Examples slight differences in ingredient temperature, slight variations in raw material quality, environmental factors effects etc.

b) **Why Process control is more preferred to product control**

Process control aims to prevent defects or errors from occurring in the first place by monitoring and controlling the variables within the process. It allows for early detection of potential issues and corrective actions to be taken before they impact the product quality as the process continues, thus avoiding losses.

In contrast, product control relies on inspecting the final product and identifying defects after they have already occurred, which can be costly and time-consuming. A lot of losses are incurred if many items are found to be defective.

c) $CL=\mu=2.0$,and control limits are given by

$$\mu \pm 3\frac{\sigma}{\sqrt{n}}=2.0\pm 3\times\frac{.25}{\sqrt{5}}=2.0\pm 0.3354$$

$UCL=2.3354 \wedge LCL=1.6646$

Probability that a process is out of control = 1- probability that the process is under control

$$¿1-P\left(1.6646\leq\overline{X}\leq 2.3354\right)$$

$$¿1-P\left(\frac{1.6646-2.0}{0.25/\sqrt{5}}\leq Z\leq\frac{2.3354-2.0}{0.25/\sqrt{5}}\right)$$

$$¿1-P\left(-3.0\leq Z\leq 3.0\right)$$

$$¿1-\{\phi(3)-\phi(-3)\}$$

$$¿1-0.0026=0.9974\,0r\,99.74\,\%$$

d) (i) C chart because the proportion of defective cannot be computed since the total number of units inspected is unknown.

(ii)

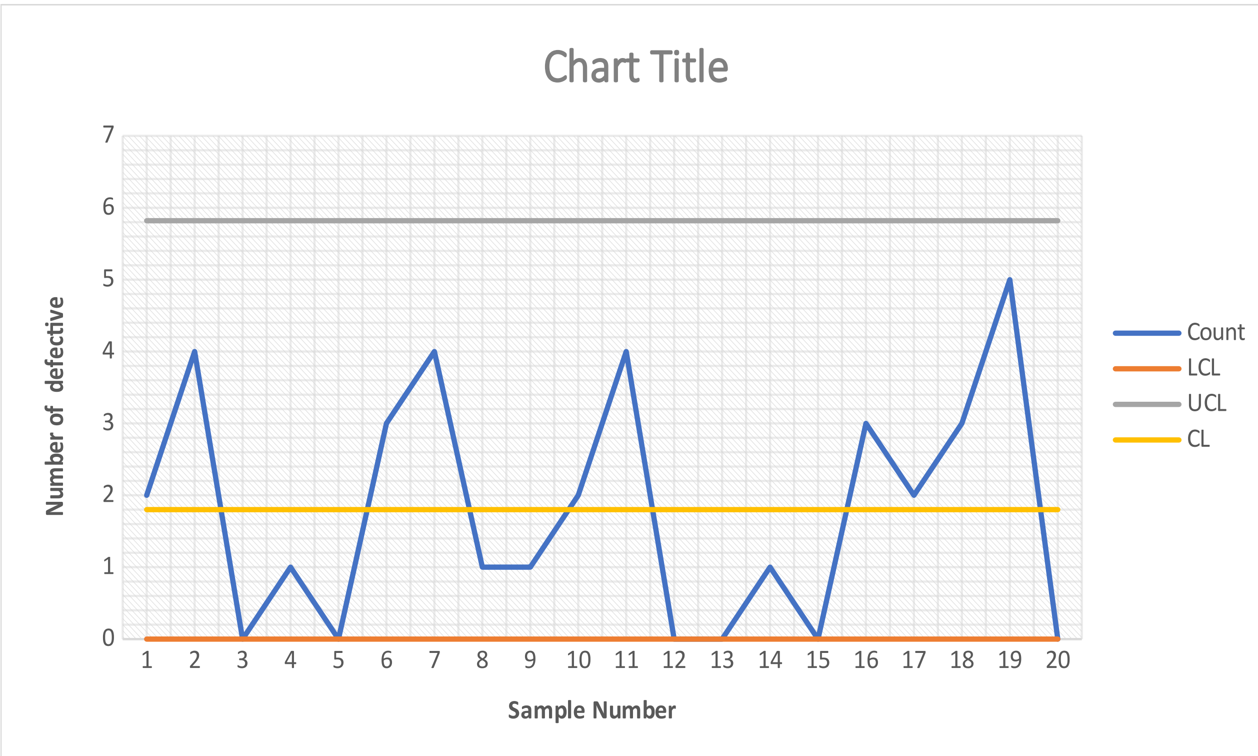
$$CL = \overline{C} = \frac{2+4+0+1+\ldots+0}{20} = \frac{36}{20} = 1.8$$

Control limits are then given by

$$\overline{C} \pm 3\sqrt{\overline{C}} = 1.8 \pm 3 \times \sqrt{1.8} = 1.8 \pm 4.0249$$

$$LCL = 1.8 - 4.0249 = -2.2249 \cong 0$$

$$UCL = 1.8 + 4.0249 = 5.8249$$



Since of the points lie outside the control limits, the process is in control.

e)

$$CL = \overline{p} = \frac{\textit{Total number defects}}{\textit{Total Number of Units sampled}} = \frac{6148}{20 \times 2000} = 0.1537$$

Control limits are given by

$$\overline{p} \pm 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.1537 \pm 3\sqrt{\frac{0.1537(1-0.1537)}{200}}$$

$$¿ 0.1537 \pm 0.0765$$

$$LCL = 0.1537 - 0.0765 = 0.0772$$
$$UCL = 0.1537 + 0.0765 = 0.2302$$

The proportions of defective for each sample is given as

$$P_i = \frac{d_i}{n} = \frac{d_i}{200}$$

The results are as given in the table below.

0.21	0.22	0.11	0.17	0.11	0.16	0.14	0.15	0.17	0.15
0.18	0.2	0.11	0.13	0.06	0.2	0.1	0.16	0.14	0.19

The proportion of sample number 15 is below $LCl = 0.077$. The process is out of control

f) $n = 200$, $c = 4$, $AQL = 0.5$ percent, and $LTPD = 4$ percent,

The distribution is

$$P(D = d) = \begin{cases} \binom{200}{d} p^d (1-p)^{200-d}, d = 0, 1, 2, \ldots, 200 \\ w\, 0, Otherwise \end{cases}$$

Producer's risk is the probability of rejecting a lot that meets AOQ

$$\beta = 1 - P(D \leq 4 \vee good\, lot) = 1 - \sum_{d=0}^{4} \binom{200}{d} (0.005^d)(0.995)^{200-d} = 1 - 0.9813 = 0.0187$$

Consumer's risk is the probability of accepting a lot doesn't meeting LTPD

$$\alpha = P(D \leq 4 \vee bad\, lot) = \sum_{d=0}^{4} \binom{200}{d} (0.96^d)(0.04)^{200-d} = 0.09502$$

QUESTION TWO (20 MARKS)

a) Process capability refers to the ability of a process to consistently produce output that meets the specified requirements or targets.

b) *Control limits are given by* 45.0 ± 3.0 thus

$$UCL = 48.0 \wedge LCL = 42$$

$$C_p = \frac{UCL - LCL}{6\sigma}$$

$$¿ \frac{48 - 42}{6 \times 0.9} = \frac{6}{5.4} = 1.1111$$

$$C_{pk} = min\left(\frac{UCL - \mu}{3\sigma}, \frac{\mu - LCL}{3\sigma}\right)$$

$$¿ min\left(\frac{48 - 45.5}{3 \times 0.9}, \frac{45.5 - 42}{3 \times 0.9}\right)$$

$$¿ min\left(\frac{2.5}{2.7}, \frac{2.5}{2.7}\right) = 0.92592$$

Cp > 1 but Cpk < 1: In this case, although the overall process spread is within the specification limits (Cp > 1), the process mean is not well-centered within the specification range (Cpk < 1). This suggests a potential shift or bias in the process mean. Although the process is still capable of meeting the specifications, there is a risk of producing more defects or non-conforming items due to the offset from the target value.

c) Probability of accepting a lot with various proportion of defective (p) is computed as

$$p_a = P(accepting\,lot) = \sum_{d=0}^{1} \binom{12}{d} p^d (1-p)^{12-d} \vee obtained \in the\,table\,provided$$

AOQ is hen computed as

$$AOQ = \frac{P(N-n)P_a}{n}$$

The values are shown below

Proportion of defective	np	Probability of accepting	AOQ
0	0	1	0
0.01	0.12	0.994	0.009344
0.02	0.24	0.977	0.018368
0.04	0.48	0.919	0.034554
0.06	0.72	0.84	0.047376
0.08	0.96	0.751	0.056475
0.1	1.2	0.659	0.061946
0.12	1.44	0.569	0.064183
0.14	1.68	0.483	0.063563
0.16	1.92	0.405	0.060912
0.18	2.16	0.336	0.056851
0.2	2.4	0.275	0.0517
0.3	3.6	0.085	0.02397
0.4	4.8	0.0196	0.00737
0.5	6	0.003	0.00141

The curve is then fitted as follows

QUESTION THREE (20 MARKS)

a)

i) **Single Sampling Plan:** The inspection process involves taking the sample, checking each item for defects, counting the number of defective items, and then comparing the number of defects with the acceptance number (c). If the number of defects in the sample is less than or equal to c, the entire lot is accepted; otherwise, the lot is rejected.

ii) **Double Sampling Plan**

The decision process in Double Sampling involves inspecting the first sample. If the number of defects is within the first acceptance number (c1), the lot is accepted immediately. If the number of defects exceeds c1, a second sample is taken and inspected. Based on the results of the second sample, a final decision is made on whether to accept or reject the lot.

iii) **Sequential Sampling Plan**

It is a dynamic acceptance sampling plan that involves a sequential inspection process. Instead of inspecting a fixed sample size all at once, items are inspected one by one, and the decision to accept or reject the lot is made sequentially based on the observed number of defects.

b) $N=400, n=15, c=0, AQL=0.01, LTPD=0.10$

The distribution is

$$P(D=d)=\begin{cases}\binom{15}{d}p^d(1-p)^{15-d}, d=0,1,2,\ldots,15 \\ w0, Otherwise\end{cases}$$

Producer's risk is the probability of rejecting a lot that meets AOQ

$$\alpha=1-P(D=0 \vee good\ lot)=1-\sum_{d=0}^{0}\binom{15}{d}(0.01^d)(0.99)^{15-d}$$

$¿1-0.0860=0.1399$

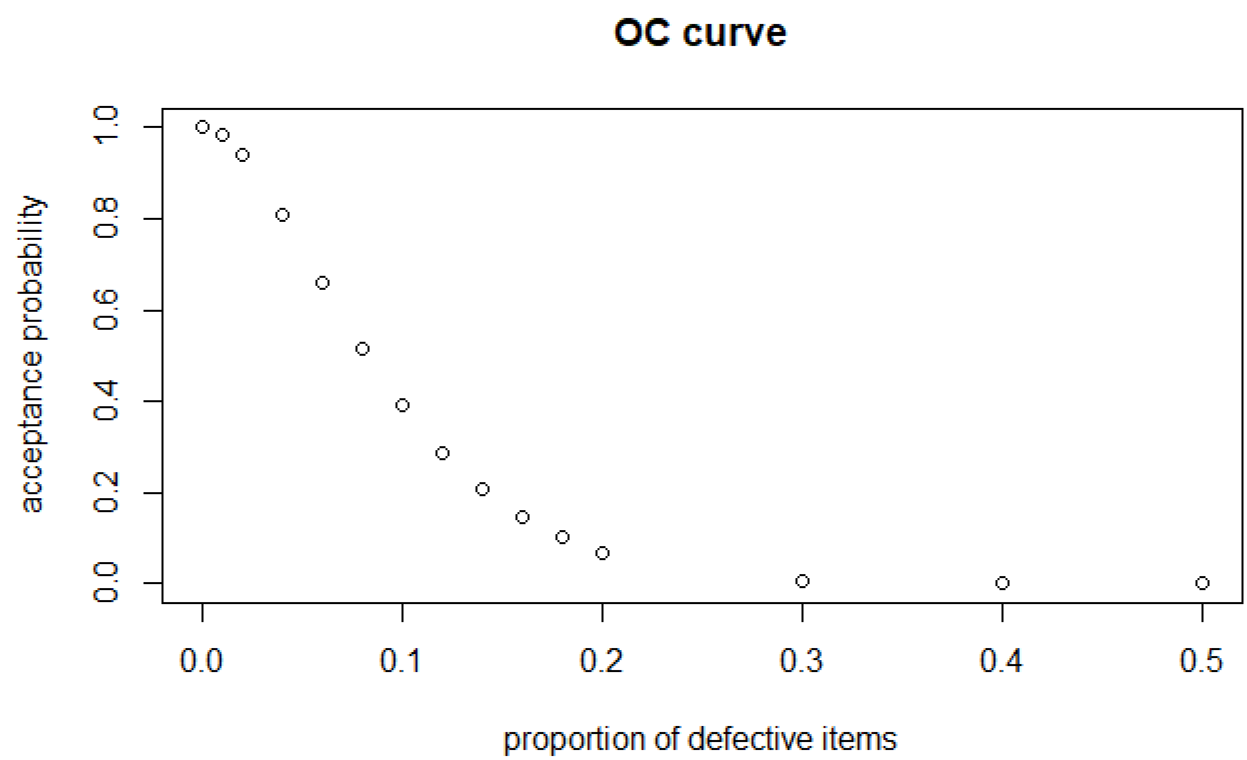
Consumer's risk is the probability of accepting a lot doesn't meeting LTPD

$$\beta=P(D\leq 0 \vee bad\ lot)=\sum_{d=0}^{0}\binom{15}{d}(0.1^d)(0.9)^{15-d}=0.2059$$

c) Probability of accepting a lot with various proportion of defective (p) is computed as

$$p_a=P(accepting\ lot)=\sum_{d=0}^{1}\binom{20}{d}p^d(1-p)^{20-d} \vee obtained \in the\ table\ provided$$

Proportion of defective	np	Probability of accepting
0	0	1
0.01	0.12	0.983
0.02	0.24	0.940
0.04	0.48	0.810
0.06	0.72	0.660
0.08	0.96	0.517
0.1	1.2	0.392
0.12	1.44	0.289
0.14	1.68	0.208
0.16	1.92	0.147
0.18	2.16	0.102
0.2	2.4	0.069
0.3	3.6	0.008
0.4	4.8	0.001
0.5	6	0.000



QUESTION FOUR (2O MARKS)

SN		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Sample Values	**1**	-1	2	1	2	1	1	-1	1	2	-2	0	2	0	0	1	2	2	0	3	-1
	2	2	0	1	1	-1	-1	1	1	1	1	1	1	1	0	-1	1	0	2	-3	2
	3	1	1	0	0	0	2	0	2	-1	-2	-3	-1	-3	-1	2	-1	1	1	-1	1
	4	0	0	0	1	0	0	-2	-1	0	2	2	0	2	0	0	0	0	-1	1	1
	5	1	1	1	0	-1	-2	1	0	0	1	1	0	1	1	-2	0	1	1	2	2
Average (in $\times 10^{-1}$¿		6	8	6	8	-2	0	-2	6	4	0	2	4	2	0	0	4	8	6	4	10
Range		3	2	1	2	2	4	3	3	3	4	5	3	5	2	4	3	2	3	6	3

For $X - \bar{¿}$ chart

$$CL = \overline{X} = \frac{1}{L}\sum \overline{X}_i = \frac{7.4}{20} = 0.37$$

$$\overline{R} = \frac{1}{L}\sum R_i = \frac{63}{20} = 3.15$$

$$UCL = \overline{X} + A_2\overline{R} = 0.37 + .577 \times 3.15 = 2.19$$

$$UCL = \overline{X} - A_2\overline{R} = 0.37 - .577 \times 3.15 = -1.45$$

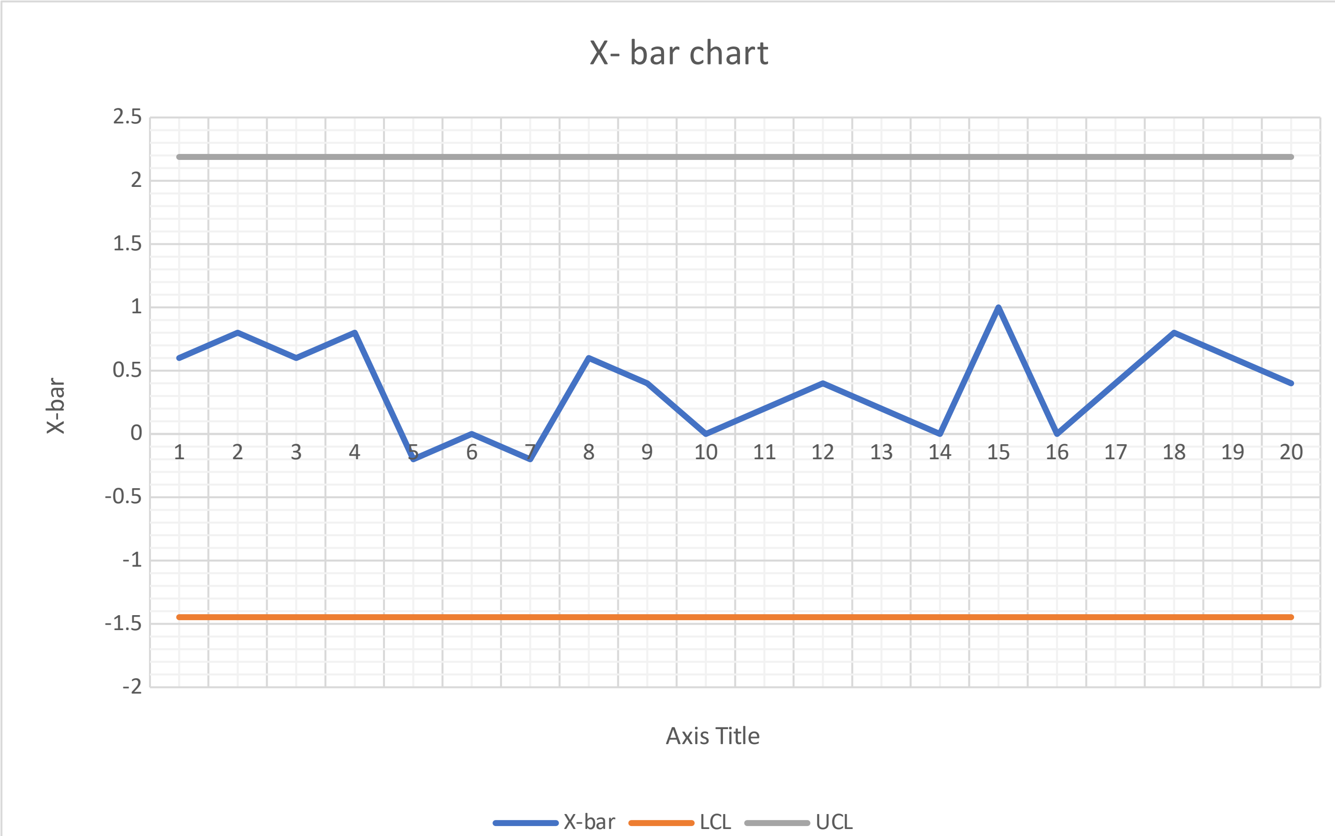
For R chart

$$CL = \overline{R} = 3.15$$

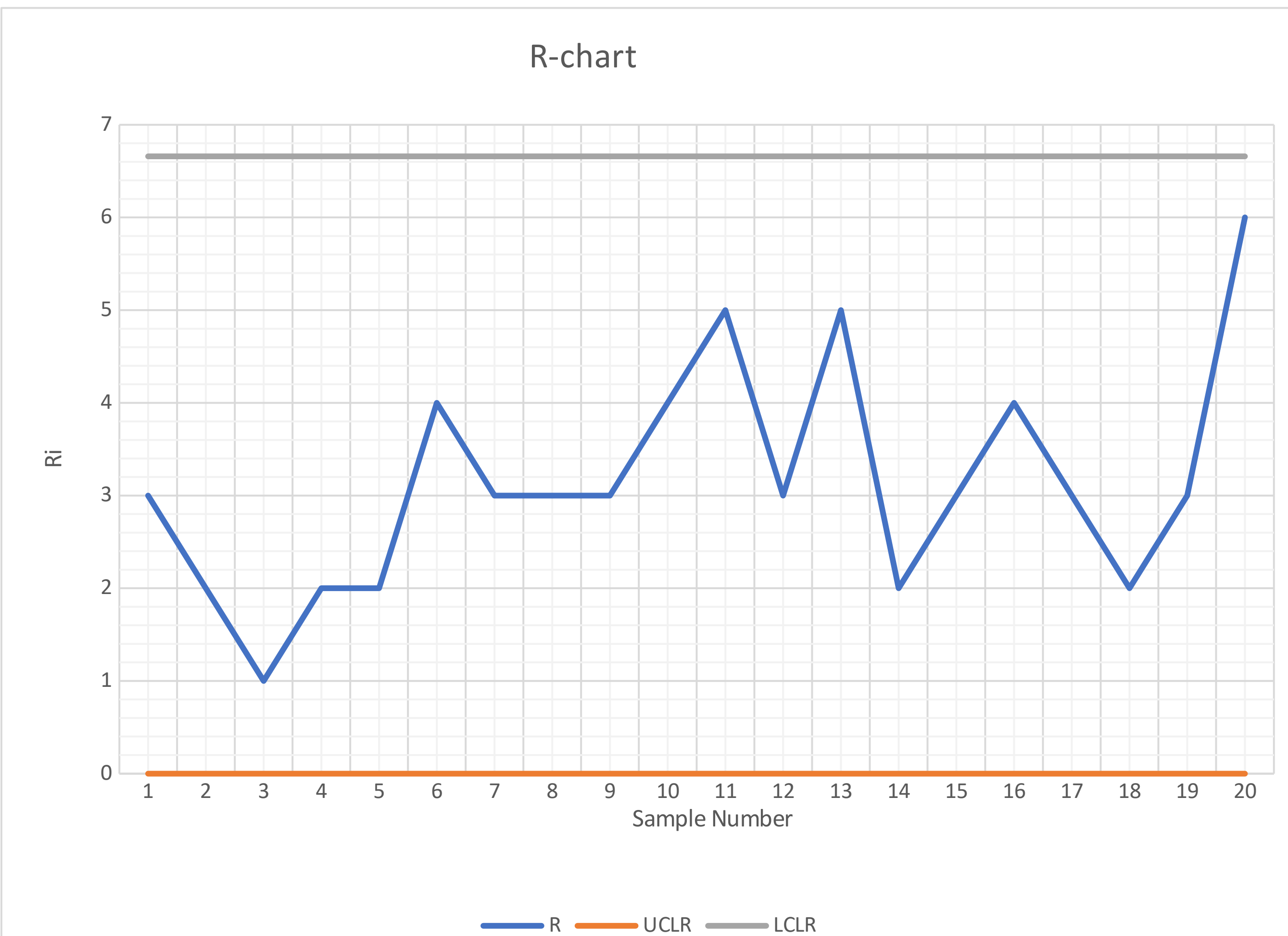
$$D_4 = 2.115 \wedge D_3 = 0$$

$$UCL = D_4\overline{R} = 2.115 \times 3.15 = 6.66$$

$LCL = D_3\overline{R} = 0$The charts are as shown below



X- bar chart
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
X-bar
Axis Title
X-bar
LCL
UCL



R-chart
7
6
5
4
3
2
1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Ri
Sample Number
R
UCLR
LCLR

QUESTION FIVE (20 MARKS)

a)

b)

$$CL = \bar{p} = \frac{total\,defective}{Total\,item\,sampled} = \frac{85}{10 \times 100} = 0.085$$

$$np = 100 \times 0.085 = 8.5$$

$$se(n\bar{p}) = \sqrt{n\bar{p} - (1 - \bar{p})} = \sqrt{8.5 - (1 - 0.085)} = 2.7541$$

Thus, control limits are given by

$$UCL = 8.5 + 3 \times 2.7541 = 16.76 \wedge LCL = 8.5 - 3 \times 2.7541 = 0.24$$