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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2016/2017 ACADEMIC YEAR
THIRD YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS)

Date: 15th August, 2016.
Time: 11.00am – 1.00pm

KMA 312 - OPERATIONS RESEARCH 1

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Write the following linear program in standard form;

i) Min $Z = 4X_1 - 2X_2 + 3X_3$

$subject\ to$: $5X_1 + 2X_2 - 3X_3 \geq -8$

$2X_1 - 2X_2 + X_3 \leq 9$

$X_1 \geq 0$

(4 Marks)

ii) Max $Z = 2X_1 + 3X_2 + 5X_3$

$subject\ to$: $X_1 + X_2 - X_3 \geq -5$

$-6X_1 + 7X_2 - 9X_3 \leq 4$

$X_1 + X_2 + 4X_3 = 10$

$X_1, X_2, X_3 \geq 0$

(4 Marks)

b) Use simplex method to solve the following linear program.

Max $Z = 2X_1 - X_2$

Subject to: $X_1 - 2X_2 \leq 2$

$3X_1 - 2X_2 \leq 18$

$X_1, X_2 \geq 0$

(8 Marks)

c) The products of two plants A and B are to be transported to three warehouses W_1, W_2,W_3. The cost of transportation of each unit from the plants to the warehouses along with the normal capacities of plants and warehouses are indicated in the table.

		WAREHOUSES			
		1	2	3	supply
PLANTS	A	25	17	25	300
	B	15	10	18	500
	demand	300	300	500	

(7 Marks)

d) Use the dual simplex method to solve the following LP.

$$\text{Maximize} \quad Z = -X_1 - X_2 - X_3$$
$$subject\ to: \quad X_1 + 2X_2 + 4X_3 \geq 2$$
$$2X_1 + X_2 + 5X_3 \leq 3$$
$$X_1 + 2X_2 + 3X_3 \leq 3$$
$$X_1, X_2, X_3 \geq 0$$

(7 Marks)

QUESTION TWO (20 MARKS)

a) State the complementary slackness principle.

(2 Marks)

b) Consider the linear program;

$$\text{Min} \quad Z = 10X_1 + 6X_2 + 8X_3$$

$$\text{Subject to:} \quad X_1 + X_2 + 2X_3 \geq 2$$
$$5X_1 + 3X_2 + 2X_3 \geq 1$$
$$X_1, X_2, X_3 \geq 0$$

i) Write the dual model for this program.

(4 Marks)

ii) Solve the dual and primal programs by the simplex method and thereby illustrate the complementary slackness principle using the final tableaus.

(14 Marks)

QUESTION THREE (20 MARKS)

a) Use the two-phase method to solve the linear program;

$$\text{Maximize} \quad Z = 2X_1 - X_2 + X_3$$
$$subject\ to: \quad 2X_1 + 3X_2 - 5X_3 \geq 4$$
$$-X_1 + 9X_2 - X_3 \geq 3$$
$$4X_1 + 6X_2 - 3X_3 \leq 8$$
$$X_1, X_2, X_3 \geq 0$$

(12 Marks)

b) A firm can produce three types of clothes; A, B, and C. Three kinds of wool are required for it, say red, green and blue. One unit length of type A cloth requires 2 meters of red, 3 meters of blue wool; one unit length of type B cloth needs 3 meters of red wool, 2 meters of green wool and 2 meters of blue wool. While as one unit length of type C cloth needs 5 meters of green wool and 4 meters of blue wool. The firm has only a stock of 8m of red wool, 10m of green wool and 15m of blue wool. It is assumed that the income obtained from one unit length clothes of types A, B, and C are Shs. 30, Shs. 50 and Shs,40 respectively. Formulate the problem as a linear program. (**DO NOT SOLVE**)

(8 Marks)

QUESTION FOUR (20 MARKS)

a) Define the following terms;

i) Dual price

(2 Marks)

ii) Shadow price

(2 Marks)

b) A paper mill converts pulpwood to low, medium and high grade newsprint. The pulpwood requirements for each newsprint, availability of each pulpwood, and selling price (per ton) are shown below:

	Low grade	Medium grade	High grade	Available (tons)
Virginia pine	2	2	1	180
White pine	1	2	3	120
Loblolly pine	1	1	2	160
Price	900	1000	1200	

The associated linear program is;

$$\text{Maximize} \quad Z = 900X_1 + 1000X_2 + 1200X_3$$
$$subject\ to: \quad 2X_1 + 2X_2 + X_3 \leq 180$$
$$X_1 + 2X_2 + 3X_3 \leq 120$$
$$X_1 + X_2 + 2X_3 \leq 160$$
$$X_1, X_2, X_3 \geq 0$$

With the optimal tableau

Basic	x_1	x_2	x_3	x_4	x_5	x_6	solution
x_2	- 1/4	1	0	1/2	- 1/4	0	84
x_3	3/2	0	1	0	1/2	0	12
x_6	2	0	0	-2	1	1	52
Z	0	200	0	300	300	0	90,000

i) In what range can the price of low grade paper vary without changing the optimal basis?
ii) What is the new optimal solution if the price of low grade paper changes to 800?
iii) In what range can the availability of Virginia pine vary without changing the optimal basis?
iv) If 10 additional tons of Virginia pine is obtained, by how much will the optimal profit change?
v) What would the plant manger be willing to pay for an additional ton of Loblolly pine?

QUESTION FIVE (20 MARKS)

An automobile manufacturer has orders for locations 5, 6, and 7 for 75, 60 and 80 respectively. The production process consist in making the body either at location 1 or 2 then shipping the body either to location 3 or 4 where it is assembled onto the rest of the car and then shipping the entire unit to a waiting customer. Production cost per body is 533 at location1 and 550 at location 2. Assembly cost at location 3 and 4 are 2256 and 2239 respectively. Transportation costs between locations are as follows;

Location	3	4
1	45	59
2	65	52

Location	5	6	7
3	72	65	79
4	81	74	63

Production capacities at locations 1 and 2 are 150 and 170 respectively; locations 3 and 4 can assemble all the bodies forwarded to them.

i) Use a diagram to illustrate the transshipment problem
(4 Marks)

ii) Set up the initial transportation tableau.
(3 Marks)

iii) Determine a production and shipping schedule that will meet all demand at minimum cost.
(13 Marks)