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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2016/2017 ACADEMIC YEAR
FOURTH YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

Date: 10th August, 2016.
Time: 8.30am – 10.30am

KMA 300 - APPLICATIONS OF LINEAR ALGEBRA

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) A company has 4 machines available for assignment to 4 tasks. Any machine can be assigned to any task, and each task requires processing by one machine. The time required to set up each machine for the processing of each task is given in the table below. Find an allocation schedule that minimizes the total setup time needed for the processing of all four tasks.

	TIME (Hours)			
	Task 1	Task 2	Task 3	Task 4
Machine 1	13	4	7	6
Machine 2	1	11	5	4
Machine 3	6	7	2	8
Machine 4	1	3	5	9

(7 Marks)

b) The pattern of rainy and sunny days in a location is a homogeneous markov chain with two states. Every sunny day is followed by another sunny day with probability 0.8. Every rainy day is followed by another rainy day with probability of 0.6;

i) Find the transition probability matrix. (2 Marks)

ii) If today is sunny, what is the chance of rain the day after tomorrow? (2 Marks)

iii) Find the steady-state distribution of the Markov chain. (3 Marks)

c) Calculate the optimal strategy for each player, and the value of the game below. The payoff matrix in is given in the perspective of player X.

		Y 1	Y 2	Y 3
X	1	16	-8	4
X	2	-24	-16	3
X	3	1	1	2

(5 Marks)

d) Consider the closed three sector economy consisting of say Energy, Manufacturing and Services where the exchange matrix is given by matrix below. Find the production vector P.

$$A = \begin{bmatrix} 0.2 & 0.3 & 0.2 \\ 0.5 & 0.2 & 0.3 \\ 0.3 & 0.5 & 0.2 \end{bmatrix}$$

(6 Marks)

e) Find an equation of the line passing through the points (2, 1), (1, 5) and (3 given the general equation $ax + by + c = 0$

(5 Marks)

<u>QUESTION TWO (20 MARKS)</u>

a) Define a two player zero sum game and give the conditions of a competitive game.

(5 Marks)

b) Each of two players has an Ace, King, Queen and Jack cards. They each simultaneously show a card. Player 1 wins if they both show an Ace, or if neither shows an Ace and the cards do not match. Player 2 wins if exactly one shows an Ace or if neither shows and Ace and the cards match. The winner receives a payment of $1 from the loser.

i) Formulate a pay off matrix for the game described.

(4 Marks)

ii) Find the optimal strategy for each player.

(8 Marks)

iii) Find the value of the game.

(3 Marks)

<u>QUESTION THREE (20 MARKS)</u>

a) A medical researcher is studying the spread of a virus in 1000 labs mice. During any given week it is estimated that there is an 80% chance that a mouse will overcome the virus, and during the same week there is a 10% likelyhood a healthy mouse will become infected.

i) Find the transition probability matrix

(3 Marks)

ii) How many mice are sick next week?

(3 Marks)

iii) What is the steady state solution?

(5 Marks)

b) In a shop operation, four jobs may be performed on any of the four machines. The hours for each job on each machine are presented in the table below. The supervisor would like to assign jobs so that total time is minimized. Machine Z can be assigned more than one job. Find the allocation schedule that minimizes the total time.

MACHINE				
JOB	W	X	Y	Z
A12	10	14	16	13
A15	12	13	15	12
B2	9	12	12	11
B9	14	16	18	16

(10 Marks)

QUESTION FOUR (20 MARKS)

a) An economy is based on three sectors, agriculture (A), energy (E) and manufacturing (M). Production of a dollar's worth of agriculture requires an input of \$0.2 from agriculture sector and \$0.4 from the energy sector. Production of a dollar's worth of energy requires an input of \$0.2 from the energy sector and \$0.4 from the manufacturing sector. Production of a dollar's worth of manufacturing requires an input of \$0.1 from the agriculture sector, \$0.1 from the energy sector, and \$0.3 from the manufacturing sector.

i) Formulate the input-output matrix

(4 Marks)

ii) Find the output from each sector that is needed to satisfy the demand.

(8 Marks)

b) Reduce by dominance to 2×2 game and thus find the optimal strategy.

5	4	1	0
4	3	2	-1
0	-1	4	3
1	-2	1	2

(8 Marks)

QUESTION FIVE (20 MARKS)

Consider the Leslie matrix $L = \begin{bmatrix} 3 & 5 \\ 0.8 & 0 \end{bmatrix}$

a) Find the eigen values and eigenvectors for L. (4 Marks)

b) Find the growth rate λ_1 from the dominant eigen value. (5 Marks)

c) Find the stable range distribution vector . (6 Marks)

d) Given the initial population vector $N(0) = \begin{bmatrix} 100 \\ 100 \end{bmatrix}$, find c such that for large k $N(k) = (\lambda 1)^k CX_1$.
(use k =10) (5 Marks)