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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2024/2025 ACADEMIC YEAR
FIRST YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS AND COMPUTER SCIENCE)

Date: 12th April, 2024
Time: 8.30am – 10.30am

KMA 106 - PROBABILITY AND STATISTICS 1

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) Let X be a random variable with probability density function given by

$$f(x) = \begin{cases} kx(1-x), & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find;

- i) the value of the constant k . (2 marks)
 - ii) the cumulative distribution function (CDF) of X . (2 marks)
 - iii) use the CDF above to compute $P\left(\frac{1}{2} \leq X \leq 1\right)$. (2 marks)
- b) Consider a random variable X with probability distribution given by
- $$f(x) = \begin{cases} \frac{1}{32}(2x+3), & x = 1, 2, 3, 4 \\ 0, & \text{elsewhere} \end{cases}$$
- i) Determine moment generating function (mgf) of X . (3 marks)
 - ii) Use mgf to find;
 - i) Mean of X . (2 marks)
 - ii) Variance of X . (3 marks)
- c) The average rate of job submissions in a busy computer center is 4 per minute. If it can be assumed that the number of submissions per minute interval is Poisson distributed, calculate the probability that:
- i) a job will be submitted between 1 and 3 (inclusive) minutes? (3 marks)
 - ii) More than 1 minutes will elapse between jobs? (3 marks)

- d) The length of human pregnancies from conception to birth is approximately normally distribution with a mean of 266 days and a standard deviation of 16 days.
- What proportion of all pregnancies will last between 240 and 270 days (roughly between 8 and 9 months)? (3 marks)
 - What length of time marks the shortest 70% of all pregnancies? (3 marks)
- e) Boys of a certain age are known to have a mean weight of $\mu = 85$ pounds. A complaint is made that the boys living in a municipal children's home are underfed. As one bit of evidence, $n = 25$ boys (of the same age) are weighed and found to have a mean weight of $\bar{X} = 80.94$ pounds. It is known that the population standard deviation σ is 11.6 pounds. Based on the available data, what should be concluded concerning the complaint? Take $\alpha = 0.05$. (4 marks)

QUESTION TWO (20 MARKS)

- a) A random variable X has a pdf given by
- $$f(x) = \begin{cases} 6e^{-6x}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$
- Determine mgf of X . (4 marks)
 - Use mgf to find
 - Mean of X . (3 marks)
 - Variance of X . (3 marks)
- b) The lifetime of a light bulb is X hours, where X can be modelled by an exponential distribution with parameter $\lambda = 0.0125$.
- Find the mean and variance of the lifetime of a light bulb. (4 marks)
 - Out of 1000 bulbs, how many bulbs do you expect to have a lifetime that is;
 - less than 100 hours. (3 marks)
 - between 50 hours and 150 hours. (3 marks)

QUESTION THREE (20 MARKS)

- a) Let X be a Poisson random variable with the parameter λ . That is,

$$f(x) = \begin{cases} \frac{\lambda^x e^{-\lambda}}{x!}, & x = 0, 1, \dots \\ 0, & \text{elsewhere} \end{cases}$$

Without using moment generating function, find

- Mean of X . (4 marks)
 - Variance of X . (6 marks)
- b) Suppose it has been observed that, on average, 180 cars per hour pass a specified point on a particular road in the morning rush hour. Due to impending roadworks, it is estimated that congestion will occur closer to the city center if more than 5 cars pass the point in any one minute. What is the probability that;
- There is no congestion? (3 marks)
 - Congestion occurring? (2 marks)

- c) Suppose that a machine which is known to produce 1% defective components is used for a production run of 40 components. Use Poisson approximation to Binomial distribution to calculate the probability that two defective items are produced. (5 marks)

QUESTION FOUR (20 MARKS)

- a) The final exam scores in a statistics class were normally distributed with a mean of 63 and a standard deviation of five.
- i) Find the probability that a randomly selected student scores less than 50. (2 marks)
 - ii) Find the probability that a randomly selected student scored more than 65 on the exam. (3 marks)
 - iii) Find the probability that a randomly selected student scored between 55 and 75. (3 marks)
 - iv) Find the 90th percentile. (4 marks)
- b) The amount of time, in minutes, that a person must wait for a bus is uniformly distributed between 10 and 25 minutes, inclusive.
- i) Show that the cumulative distribution function is

$$F(x) = \begin{cases} 0, & x < 10 \\ \frac{x - 10}{15}, & 10 \leq x < 20 \\ 1, & x \geq 20 \end{cases}$$
 (3 marks)
 - ii) Using the obtained cumulative distribution function,
 - i) What is the probability that a person waits fewer than 12.5 minutes? (2 marks)
 - ii) What is the probability that will be between 0.5 standard deviation from the mean? (3 marks)

QUESTION FIVE(20 MARKS)

- a) Distinguish between a hypothesis and hypothesis testing. (3 marks)
- b) A manager will switch to a new technology if the production process exceeds 80 units per hour. The manager asks the company statistician to test the null hypothesis: $H_0: \mu = 80$ against the alternative hypothesis: $H_1: \mu > 80$ If there is strong evidence to reject the null hypothesis then the new technology will be adopted. Past experience has shown that the standard deviation is 8. A data set with $n = 25$ for the new technology has a sample mean of 83. Does this justify adoption of the new technology? Take $\alpha = 0.05$. (5 marks)
- c) A fertilizer mixing machine is set to give 12 kg of nitrate for every 100kg bag of fertilizer. Ten 100kg bags are examined. The percentages of nitrate are as follows: 11, 14, 13, 12, 13, 12, 13, 14, 11, 12. Is there reason to believe that the machine is defective at 5% level of significance? (6 marks)
- d) The following is the summary of Lung destructive index between the smokers and non-smokers

Group	Sample size	Mean	Standard deviation
Smokers	16	17.5	4.4711
Non-smokers	9	12.4	4.8492

Can we conclude at 5% level of significance that in general, smokers have greater lung damage than do non-smokers. (6 marks)