



Kasarani Campus
Off Thika Road
Tel. 2042692 / 3

P. O. Box 49274,

00100

NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
FIRST YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF **BACHELOR OF SCIENCE**
(MATHEMATICS**)**
KMA 104 - CALCULUS I

Date: 8th August, 2023
Time: 11.30 am – 1.30pm

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) Find domain and range for each of the following functions;
- i) $f(x) = (x-4)^2 + 5$ (2 Marks)
- ii) $f(x) = \sqrt{3x+2} - 1$ (2 Marks)
- iii) $f(x) = \frac{3}{x-2}$ (2 Marks)
- b) Evaluate the limits;
- i) $\lim_{x \rightarrow -1} \frac{\sqrt{x+2} - 1}{x+1}$ (2 Marks)
- ii) $\lim_{x \rightarrow 3} \frac{x^2 - 3x}{2x^2 - 5x - 3}$ (2 Marks)
- c) Find the derivative of the function $f(x) = x^2 - 2x$ using first principle method. (3 Marks)
- d) The position of a particle along a straight line at time t seconds is given by $s(t) = 3t^2 - 4t + 1$. Find the function that describes its acceleration at time t . (3 Marks)
- e) Given $(x) = x^2 + 1 \wedge g(x) = \frac{1}{x}$, find each of the following:
- i) $(foh)(x)$ (2 Marks)
- ii) $(gof)(x)$ (2 Marks)
- f) Differentiate the following functions:
- i) $e^x(2x^2 + 1)$ (2 Marks)
- ii) $\ln(x^2 + 2)$ (2 Marks)
- iii) $\cos^3(4\theta + 1)$ (2 Marks)
- iv) $e^{3x} \sin(2x + 1)$ (2 Marks)
- v) $y = \left(\frac{x}{3x+2}\right)^5$ (2 Marks)

QUESTION TWO (20 MARKS)

- a) Find the equation of the tangent to the curve with equation $y = \sin x$ at the point $x = \frac{\pi}{3}$. (4 Marks)
- b) Find the second derivatives of the following functions; (3 Marks)
- i) $x^2 + y^2 = 25$
- ii) $y = \ln(1 + t^4)$ (3 Marks)
- c) Find the x-coordinates of the points of inflection for the curve with equation; (4 Marks)
- $$y = \frac{x^4}{12} + \frac{x^3}{3} - \frac{3x^2}{2} + 3x - 7$$
- d) Verify that the following functions satisfy the criteria stated in Rolle's Theorem and find all values c in the given interval where $f'(c) = 0$
- i) $f(x) = x^2 + 2x$ over $[-2, 0]$ (3 Marks)
- ii) $f(x) = x^3 - 4x$ over $[-2, 2]$ (3 Marks)

QUESTION THREE (20 MARKS)

- a) Find the equation of the line tangent to the graph $f(x) = x^2 - 4x + 6$ at $x = 1$. (3 Marks)
- b) Differentiate the following function implicitly; (3 Marks)
- i) $y^2 + x^3 - y^3 + 6 = 3y$
- ii) $y^2 + x^3 - xy + \cos y = 0$ (3 Marks)
- c) A particle moves along a coordinate axis in such a way its position that at time t is given by $s(t) = 2 \sin t - t$ for $0 \leq t \leq 2\pi$. At what time is the particle at rest? (3 Marks)
- d) Find the derivatives of the following; (2 Marks)
- i) $y = \tan(4x^2 - 3x + 1)$
- ii) $y = 5x^3 \sin x$ (2 Marks)
- iii) $f(x) = \frac{\cos x}{4x^2}$ (2 Marks)
- iv) $y = \cos^4(7x^2 + 1)$ (2 Marks)

QUESTION FOUR (20 MARKS)

- a) Evaluate the limits; (3 Marks)
- i) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$
- ii) $\lim_{x \rightarrow \infty} \frac{3x^5 + x^3 - 5}{6 + x + 7x^2}$ (3 Marks)
- b) Show that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ (4 Marks)
- c) A rocket is launched so that it rises vertically. A camera is positioned 5000 feet from the launch pad. When the rocket is 100 feet above the launch pad, its velocity is 600 ft/sec. Find the rate of change of the camera's angle as a function of time so that it stays focused on the rocket. (10 Marks)

QUESTION FIVE (20 MARKS)

- a) Find the extrema of the following function $f(x) = -x^2 + 3x - 2$ over $[1, 3]$. (7 Marks)
- b) A ball is dropped from a height of 64 feet. Its height above the ground (in feet) t seconds later is given by $s(t) = -16t^2 + 64$.

i) What is the instantaneous velocity of the ball when it hits the ground? (3 Marks)

ii) What is the average velocity during its fall. (3 Marks)

c) A rocket travels in an elliptical orbit whose path is given by the equation $45x^2 + 25y^2 = 100$. The rocket can fire missiles along lines tangent to its path. If the rocket fires a missile when it is located at $(3, \frac{8}{3})$, where will it intersect the x-axis? (7 Marks)