KIRIRI WOMEN'S UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2024/2025 ACADEMIC YEAR
THIRD YEAR, SECOND SEMESTER EXAMINATION
FOR THE BACHELOR OF SCIENCE
(COMPUTER SCIENCE)

KMA 312 – OPERATIONS RESEARCH 1

Date: 19th April 2024
Time: 8:30am – 10:30am

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Define a linear programing problem (2 Marks)
   Hence or otherwise, explain the following terms as used while characterizing linear programing problems
   i) Decision variables (1 Mark)

   ii) Constraints (1 Mark)

   iii) Objective function (1 Mark)

b) State the standard form of an LPP and hence or otherwise, using slack variables, give the tableau format of the standard form. (4 Marks)

c) Orcas Research Limited has three research projects at hand to be subcontracted to four consultancy firms. Possible returns (in '000 USD) gained from assigning the consultancy firms to the research projects are given below

Consultancy Firms

		1	2	3	4
Projects	**1**	10	20	25	34
	2	9	15	25	31
	3	12	19	27	38

   i) Give the recursive formula for this resource distribution problem (2 Marks)
   ii) Find the optimal assignment that will maximize the returns gained (5 Marks)

d) Distinguish between the following terms
   i) Deterministic and probabilistic mathematical models (2 Marks)
   ii) Infeasible and unbounded solutions (2 Marks)

e) Use graphical method to find the optimal solution of the following linear programing problem

Maximize $\quad x_0 = 5x_1 + 2x_2$

Subject to: $\quad 10x_1 + 2x_2 \le 2100$

$$x_1 + 0.5x_2 \le 600$$

$$x_2 \ge 800$$

$$x_1 \ge 0, \ x_2 \ge 0$$

(5 Marks)

f) Given the following constraints, minimize the value of $z = -0.4x + 3.2y$.

$$\begin{bmatrix} x \ge 0 & x+y \le 7 \\ y \ge 0 & x+2y \ge 4 \\ x \le 5 & y \le x+5 \end{bmatrix}$$

(5 Marks)

QUESTION TWO (20 MARKS)

a) Davis & Shirtliff Company has offers from three different clients to fix some water pumps in a manufacturing plant. Each client would like the company to work for him/her on full-time basis. However, each client is willing to employ the company for as many days of the week as it is prepared to give for the fee (in dollars) shown in the table below

No. of Days	Client 1	Client 2	Client 3
1	100	125	150
2	250	250	300
3	400	375	400
4	525	500	550
5	600	625	650

i) Give the recursive formula for this resource distribution problem (2 Marks)
ii) Use the recursive formula in (i) above to evaluate possible returns from different clients when assigned varied number of days (5 Marks)
iii) Use tabular representation to show assignments for the possible returns obtained in (ii) above (3 Marks)
iv) What is the optimal assignment for this problems and the corresponding maximum income? (2 Marks)

b) A farmer can plant up to 8 acres of land with wheat and barley. He can earn $5,000 for every acre he plants with wheat and $3,000 for every acre he plants with barley. His use of a necessary pesticide is limited by federal regulations to 10 gallons for his entire 8 acres. Wheat requires 2 gallons of pesticide for every acre planted and barley requires just 1 gallon per acre.
i) Form a linear programing problem for this scenario (3 Marks)
ii) Use the formed set of constraints to show the feasible region graphically (2 Marks)
iii) Use the graph to determine the maximum profit the farmer can make (3 Marks)

QUESTION THREE (20 MARKS)

a) Consider the following linear programing problem (LPP)

Maximize $\quad x_0 = 3x_1 + 5x_2 - 2x_3$

Subject to: $\quad x_1 + 2x_2 + 2x_3 \leq 10$

$$2x_1 + 4x_2 + 3x_3 \leq 15$$

$$x_1 \geq 0, \ x_2 \geq 0, \ x_3 \geq 0$$

  i) Using slack variables, represent the LPP in tableau form and give the starting basic solution
  (3 Marks)
  ii) Use Simplex method to solve the LPP (4 Marks)
  iii) Use the solutions in (ii) above to find the optimal value x_0 (1 Mark)

b) An oil exploring company sells its products to three different categories of customers: executive class, middle class and lower class. Each product spends time (in hours) in the cabinet shop, finishing shop and crating shop as shown in the table below

Customer category	Cabinet shop	Finishing shop	Crating shop	Profit
Executive class, X_1	2	1	1	150
Middle class, X_2	1	2	1	125
Lower class, X_3	1	1	0.5	50
Available hours	16	16	10	

  i) Formulate a linear programing problem for this scenario (2 Marks)
  ii) Use slack variables to represent the LPP in tableau form and give the starting basic solution
  (3 Marks)
  iii) Use Simplex method to determine how many of each type of the product should be made to maximize the profit (5 Marks)
  iv) Use the solutions in (iii) above to find the optimal value of the objective function
  (2 Marks)

QUESTION FOUR (20 MARKS)

a) Explain the general set-up of a linear programing transportation problem with m supply points (where point i can supply at most s_i units), n demand points (where point j must receive at least d_j units of the shipped goods) and that unit produced at supply point i and shipped to demand point j incurs a variable cost of c_{ij} (5 Marks)

b) Company A has three production facilities P_1, P_2 and P_3 with production capacities 7, 10 and 18 per week of a product respectively. These units are to be shipped to 4 warehouses D_1, D_2, D_3, and D_4 with requirements of 5, 8, 7, and 15 respectively. Transportation costs (in '000) per unit between the production facilities to warehouses are given in the table below

	D_1	D_2	D_3	D_4
P_1	20	30	50	15
P_2	70	35	40	60
P_3	40	12	60	25

  i) Obtain the corresponding LPP for this transportation problem (4 Marks)
  ii) Explain the use of North West Corner Method in finding optimal solutions to transportation problems (2 Marks)
  iii) Perform iterations required to obtain the solution of the transportation problem
  (5 Marks)
  iv) Find the allocation that minimizes the total transportation cost (2 Marks)
  v) Obtain the minimum total transportation cost (2 Marks)

QUESTION FIVE (20 MARKS)

a) A company manufactures two products M and N. Both products require four raw materials to be input into the manufacturing process. The amount of each material required by the products is given below

Material Requirement (kg)	Product M	Product N
W	15	7.5
X	20	40
Y	15	15
Z	15	20

The availability of materials W and X is limited to 900kg and 2800kg per month respectively. The supply of the other two materials is not restricted. Management of the company has decided that no more than 80 units of product M and 60 units of product N will be produced per month and that the total monthly production of the two products will be limited to 80 units. Per unit contribution earned by M and N is $24 and $18 respectively.

   i) Formulate the LPP that maximizes the contribution

   (2 Marks)

   ii) Use graphical method to obtain the feasible region

   (4 Marks)

   iii) State the optimum production mix and calculate the associated production contribution

   (2 Marks)

   iv) Comment upon how the solution would change if the contribution per unit for product M fell to $20 and the contribution per unit for product N increased to $22

   (2 Marks)

b) Linear programing problems can be expressed in different forms including canonical form.
   i) Give the canonical form of a LPP

   (2 Marks)

   ii) State the main characteristic of canonical form

   (3 Marks)

   iii) Explain the five elementary transformations that can be used to convert any LPP into canonical form   (5 Marks)