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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR FIRST YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

> Date: 13th December, 2023 Time: 8.30am –10.30am

KMA 102 - INTRODUCTION TO PROBABILITY AND STATISTICS

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) The following are the marks (out of 100) of 60 students in mathematics.
 - 16, 13, 5, 80, 86, 7, 51, 48, 24, 56, 70, 19, 61, 17, 16, 36, 34, 42, 34, 35, 72, 55, 75,31, 52, 28,72, 97, 74, 45, 62, 68, 86, 35, 85, 36, 81, 75, 55, 26, 95, 31, 7, 78, 92, 62,52, 56, 15, 63,25, 36, 54, 44, 47, 27, 72, 17, 4, 30.
 - i) Using class interval of 10 i.e. 0-9, 10-19 etc. construct a grouped frequency distribution table (3 marks)
 - ii) Construct a histogram for the data

(3 marks)

- b) Given the following data $\sum X = 50, \sum Y = -30, \sum X^2 = 290, \sum Y^2 = 300, \sum XY = -115 \text{ and n} = 10.$
 - i) Find the equation of the regression of Y on X

(3 marks)

ii) Calculate the coefficient of correlation between X and Y

(3 marks)

In a city there are 3 stores each having 20 pieces of an item. Let these stores be denoted by S_1 , S_2 and S_3 . The stores S_1 , S_2 and S_3 . have 10%, 20% and 30% defective items, respectively. A costumer first chooses a store randomly and then selects an item randomly from the store. Find the probability that the selected item is defective.

(3 marks)

d) Let X be a continuous random variable. Show that the function

$$f(x) = \frac{1}{2}x, \ 0 \le x \le 2$$

0, otherwise

is a p.d.f of X. Hence calculate $P\left(\frac{1}{2} \le X \le 1\right)$

(3 marks)

e) Given the on marks of students in the following data.

> Marks 10 - 2020 - 30 $30 - 40 \quad 40 - 50 \quad 50 - 60 \quad 60 - 70 \quad 70 - 80$ Number of Students 7 10 10 20 20 15 8

Obtain:

- Mean (3 marks) i) Mode (2 marks) ii) iii) Median (2 marks) iv) Standard deviation (2 marks)
- f) The probability of the number of events occurring in a given period of time is known as poisson distribution. Telephone calls enter a college switchboard on the average of two every three minutes. What is the probability of 5 or more calls arriving in a 9-minute period?

(3 marks)

QUESTION TWO (20 MARKS)

Consider the following data for a group of ten students showing the number of minutes each was a) late for Mathematics lectures in a semester

6 7 5 8 14 6 5 4 6 9

Calculate the second, third and fourth central moments. i)

(5 marks)

- Using the results in (i) above obtain the Skewness and Kurtosis of this data and comment on ii) your results. (5 marks)
- The data given below represents the frequency distribution of the various shoe sizes purchased by b) customers during the first quarter of the year 2023

Class interval	5-6	6-7	7-8	8-9	9-10	10-11
frequency	40	56	60	96	84	68

Compute the

- 20^{th} percentile, P_{20} , i) (3 marks)
- The coefficient of quartile deviation (5 marks) ii)
- iii) Interpret the value obtained in (ii) above (2 mark)

QUESTION THREE (20 MARKS)

A mathematics lecturer recorded the length of time (y) minutes, taken to travel to school when a) leaving x minutes after 6 am on seven selected mornings. The result are as follows.

X	0	10	20	30	40	50	60
у	16	27	28	39	39	48	51

i) Plot a scatterplot of length of time, y, against leaving x minutes after 6am

(3 marks)

ii) Calculate the equation of the least square regression line of y on x.

(6 marks)

iii) Estimate the length of time in y minutes given x = 35

(2 marks)

b) Let X be a discrete random variable whose probability distribution is given by f(x) = kx, x = 2,3,4,5,6

0. Otherwise

i) Obtain the value of the constant k.

(3 marks)

ii) Calculate $P(3 \le X \le 6)$.

(3 marks)

iii) Compute E[5X].

(3 marks)

QUESTION FOUR (20 MARKS)

The mean of the following frequency distribution is 52 and the frequency for the class interval 30 - 40 is f.

Class interval	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Frequency	5	3	f	7	2	6	13

Find:

v)

i) Show that the value of f is 4.

(2 marks)

ii) The Mode

(3 marks)

iii) The Standard deviation

(5 marks)

iv) The 7th decile

(3 marks) (3 marks)

vi) Semi-interquartile range

The 72nd percentile

(4 marks)

QUESTION FIVE (20 MARKS)

a) Let A and B be events with $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $(A \cup B) = \frac{1}{2}$.

Find

i) P(A|B),

(3 marks)

ii) P(B|A),

(2 marks)

iii) $P(A|B^c)$.

(4 marks)

b) Given a set of data; 2, 9, 8, 4, 7, 6, 8

calculate the harmonic and the geometric mean for the data.

(6 marks)

- c) A bag contains 5 red balls and 3 black balls. If 3 balls are drawn without replacement, what is the probability that
 - i) no black balls will be selected,

(2 marks)

ii) exactly one red ball will be selected.

(3 marks)