



Kasarani Campus
Off Thika Road
Tel. 2042692 / 3

P. O. Box 49274,

00100

NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
SECOND YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN
MATHEMATICS

KMA 207: THEORY OF ESTIMATION

Date: 14th August, 2023

Time: 11.30am – 1.30pm

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) Distinguish between;
- i) Consistency and Efficiency. (2 Marks)
- ii) Point and Interval estimator. (2 Marks)

- b) Consider a random variable X with pdf

$$f(x, \theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

- i) Find the moment estimator of θ . (4 Marks)
- ii) Show that the moment estimator obtained in i) is unbiased. (2 Marks)

- c) Let x_1, x_2, \dots, x_n be a random sample of size n from be a random variable X with p.m.f

$$f(x, p) = \begin{cases} p^x (1-p)^{1-x}, & x = 0, 1 \\ 0, & \text{Otherwise} \end{cases}$$

Determine the sufficient statistic for p . (5 Marks)

- d) Let x_1, x_2, \dots, x_n be a random sample of size n from be a random variable X with a normal density

$$f(x, \mu) = \begin{cases} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2}, & -\infty < x < \infty \\ 0, & \text{Otherwise} \end{cases}$$

Find the maximum likelihood estimator of μ . (5 Marks)

- e) Consider a random sample $y_1, y_2, y_3, \dots, y_n$ of size n from Y with pdf

$$f(y, \alpha) = \begin{cases} \alpha e^{-\alpha y}, & y > 0 \\ 0, & \text{Otherwise} \end{cases}$$

Find the Cramer-Rao lower bound of $\psi(\alpha) = \frac{1}{\alpha}$. (5 Marks)

- f) To compare customer satisfaction levels of two competing cable television companies, 174 customers of Company 1 and 355 customers of Company 2 were randomly selected and were asked to rate their cable companies on a five-point scale, with 1 being least satisfied and 5 most satisfied. The survey results are summarized in the following table below;

Company 1	Company II
$n_1 = 174$	$n_2 = 355$
$\bar{x}_1 = 3.51$	$\bar{x}_2 = 3.24$
$s_1 = 0.51$	$s_2 = 0.52$

Construct a point estimate and a 99% confidence interval for $\mu_1 - \mu_2$, the difference in average satisfaction levels of customers of the two companies as measured on this five-point scale.

(5 Marks)

QUESTION TWO (20 MARKS)

- a) State the three Cramer-Rao regular conditions. (3 Marks)
 b) Show that under the regular conditions above, the Cramer-Rao inequality is given by

$$\text{Var}(T) \geq \frac{[\psi'(\theta)]^2}{I(\theta)}$$

Where $I(\theta) = E\left(\frac{\partial \log L}{\partial \theta}\right)^2 = -E\left(\frac{\partial^2 \log L}{\partial^2 \theta}\right)$, $\psi(\theta)$ is any function of θ and T is unbiased estimator of $\psi(\theta)$. (11 Marks)

- c) Let x_1, x_2, \dots, x_n be a random sample of size n from be a random variable X with a normal density

$$f(x, \mu) = \begin{cases} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, & -\infty < x < \infty \\ 0, & \text{Otherwise} \end{cases}$$

- i) Find the Cramer-Rao lower bound of $\psi(\mu) = \mu$. (4 Marks)
 ii) Hence find the UMVUE of μ , if it exists. (2 Marks)

QUESTION THREE (20 MARKS)

- a) Let x_1, x_2, \dots, x_m be a random sample of size m from $X \sim N(\mu_1, \sigma_1^2)$, where both parameters are unknown. Let y_1, y_2, \dots, y_n be another independent random sample from $Y \sim N(\mu_2, \sigma_2^2)$, also both parameters are unknown. Derive $100(1-\alpha)\%$ confidence intervals for the difference in population means $\mu_1 - \mu_2$, where both X and Y are independent variables. (10 Marks)
 b) Two independent random samples were obtained from two independent random variables $X \sim N(\mu_1, \sigma^2)$ and $Y \sim N(\mu_2, \sigma^2)$, that is $\sigma_1^2 = \sigma_2^2 = \sigma^2$ but unknown. The observations are as follows;
 X: 20, 33, 57, 22, 44, 31, 33, 40
 Y: 44, 55, 36, 65, 38, 45, 54, 50, 48, 62
 Obtain 99% confidence intervals for the difference in the two population means. (10 Marks)

QUESTION FOUR (20 MARKS)

- a) A random sample of size 10 had a mean $\bar{X} = 20$ and a standard deviation $s = 18$. Obtain 95% confidence intervals for true population variance σ^2 . (5 Marks)
 b) A random variable X has a pdf given by the gamma density

$$f(x; \alpha, \beta) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

- i) Find the moment estimators of α and β . (10 Marks)
 ii) The failure time in years of a certain machine observed over time are;

0.41, 0.58, 0.75, 0.85, 1.00, 1.08, 1.17, 1.25, 1.35

If this failure time can be model using a gamma distribution above, determine the moment estimates of $\alpha \wedge \beta$. (5 Marks)

QUESTION FIVE (20 MARKS)

a) Let $X = (x_1, x_2, \dots, x_n)$ be a random sample of size n from be a random variable X with p.m.f

$$f(x, p) = \begin{cases} p^x(1-p)^{1-x}, & x=0,1 \\ 0, & \text{Otherwise} \end{cases}$$

Find;

- i) The joint probability distribution $P(X, T)$. (4 Marks)
- ii) The distribution of the statistic T . (4 Marks)
- iii) The conditional probability distribution $P(X/T)$, hence show that $T = \sum x_i$ is sufficient for p . (3 Marks)

b) A response variable Y is related with two variables X_1 and X_2 in the form $Y = a_0 + a_2X_1 + e_i$. Data on seven sampled items are as shown in the table below;

Y	12	22	17	15	21	23	25
X_1	5	8	7	6	8	9	11

- i) Use matrix notation to fit the given linear model. (7 Marks)
- ii) Estimate the variance of each parameter given that $e_i \sim N(0,1)$. (2 Marks)