Kasarani Campus
Off Thika Road
Tel. 2042692 / 3
P. O. Box 49274, 00100
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2020/2021 ACADEMIC YEAR
SECOND YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF EDUCATION ARTS

Date: 11th December, 2020
Time: 11.30am – 1.30pm

KMA 2204 - LINEAR ALGEBRA 11

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) By reducing the given system of linear equations to its echelon form, determine the values of x, y and z

$$2x + y + z = 1$$
$$-x + 2y - 3z = 3$$
$$x + 3y - 2z = 4$$

(5 Marks)

b) Is $-1 + x^2$ in the span of $p = 1 + x + x^3$ and $q = -x - x^2 - x^3$ in P_3?

(4 Marks)

c) Show that $w = \{(x, y, 2): x, y \in R\}$ is not a subspace for R^3.

(2 Marks)

d) Find the values of k for which the matrix $T = \begin{bmatrix} k-3 & 4 \\ k & k+2 \end{bmatrix}$ has no inverse. (5 Marks)

e) Determine if b is a linear combination of the vectors

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}, u_3 = \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}, where\ b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}.$$

(5 Marks)

f) Determine whether the set of vectors $\{(3, 1, 1), (2, -1, 5), (4, 0, -3)\}$ is linearly dependent in IR^3.

(4 Marks)

g) Prove that $(AB)^{-1} = B^{-1}A^{-1}$ and hence verify using the matrices

$A = \begin{bmatrix} 1 & -3 \\ 0 & -1 \end{bmatrix}$ and B= $\begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}$ (5 Marks)

QUESTION TWO (20 MARKS)

a) Is the set T=$\{(1,1,1),(2,1,-1),(1,0,-2)\}$ a basis for $\mathbb{R}^3$? (6 Marks)

b) Solve the linear system

$$5x_1 - 2x_2 + x_3 = 1$$

$$3x_1 - 2x_2 = 3$$

$$x_1 + x_2 - x_3 = 0$$

using Cramer's rule. (6 Marks)

b) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$$

by cofactor expansion method. (8 Marks)

QUESTION THREE (20 MARKS)

a) Determine the dimension of and a basis for the solution space for the system

$$x + y + z = 0$$

$$3x + 2y - 2z = 0$$

$$4x + 3y - z = 0$$

$$6x + 5y + z = 0$$

(9 Marks)

b) Determine whether the set $S = \{2 + x + x^2, x - 2x^2, 2 + 3x - x^2\}$ is linearly independent in P_2.

(6 Marks)

c) Find the value of k for which the following matrix A is singular.

$$A = \begin{bmatrix} 1 & 2 & k \\ 3 & -1 & 1 \\ 5 & 3 & 5 \end{bmatrix}$$

(5 Marks)

QUESTION FOUR (20 MARKS)

a) Determine the value of $'a'$ so that

$$x_1 - 3x_3 = -3$$
$$2x_1 + ax_2 - x_3 = -2$$
$$x_1 + 2x_2 + ax_3 = 1$$

has

i) No solution (4 Marks)

ii) Unique solution (2 Marks)

iii) Many solution. (2 Marks)

b) Solve using Cramer's rule

$$2x + y - 2z = 10$$
$$3x + 2y + 3z = 1$$
$$5x + 4y + 3z = 4$$

(5 Marks)

c) Prove that if A is an $n \times n$ matrix which is invertible, then for any vector $\bar{b}$ in R^n the linear system $A\underline{x} = \underline{b}$ has a unique solution $A^{-1}\,b$. Hence find a unique solution for the system

$$3x + 2y + 3z = 1$$
$$2x - 2y + 4z = 6$$
$$4x + 5y - z = -2$$

(7 Marks)

QUESTION FIVE (20 MARKS)

a) Express the vector $\underline{u} = (0,1,2)$ as a linear combination of the vectors $\underline{v_1} = (-1,1,0), \underline{v_2} = (2,0,1), \underline{v_3} = (1,1,1)$.

(7 Marks)

b) Show that $S = \{(1,2,1),(2,9,0),(3,3,4)\}$ is a basis for R^3.

(7 Marks)

c) Find the basis and dimension of the solution space for the equations

$$2x_1 + 2x_2 - x_3 + x_5 = 0$$
$$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$$
$$x_1 + x_2 - 2x_3 - x_5 = 0$$
$$x_3 + x_4 + x_5 = 0$$

(6 Marks)