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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2024/2025ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS AND COMPUTER SCIENCE)

Date: 11th April, 2024
Time: 8.30am –10.30am

KMA 203 - PROBABILITY AND STATISTICS 11

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) Let X and Y be two continuous random variables. State the conditions that must be satisfied by $f(x,y)$ for it to be a joint probability density function of X and Y. (2 marks)

b) Consider bivariate random variables X and Y with joint probability density function

$$f(x,y) = \begin{cases} k(2-x)(1-y), & 0 \leq x \leq 2, \quad 0 < y < 1 \\ 0, & \text{Otherwise} \end{cases}$$

Find;

i) The value of the constant k. (3 marks)

ii) Marginal distributions of X and Y. (4 marks)

iii) Conditional distribution Y given $X = x$. (2 marks)

iv) Are X and Y independent? Give a reason for your answer. (1 mark)

c) Let X and Y be bivariate discrete random variables with probability distribution given in the table below

$f(x,y)$		Y			$f_1(x)$
		-1	**0**	**1**	
X	**0**	0.1	0.18	0.12	**0.4**
	1	0.15	0.2	0.1	**0.45**
	2	0.1	0.05	0	**0.15**
$f_2(y)$		**0.35**	**0.43**	**0.22**	

Determine;

i) Expected values of X and Y, hence write down the mean vector $\underline{\mu}$. (3 marks)

ii) Variance of X and Y. (4 marks)

iii) Covariance between X and Y hence write the covariance matrix $\mathbf{\Sigma}$. (3 marks)

iv) Correlation between X and Y. (2 marks)

d) Random variables X and Y have a bivariate normal distribution with parameters $\mu_x = 3,\ \mu_y = 5, \sigma_x^2 = 4,\ \sigma_y^2 = 9$ and $\sigma_{xy} = 3$.

i) Write down the joint probability distribution of X and Y. (4 marks)

ii) Obtain the conditional expectation and conditional variance of X given Y=4. (2 marks)

QUESTION TWO (20 MARKS)

The joint probability distribution of X and Y is as shown below.

$$f(x,y) = \begin{cases} e^{-x-y}, & x > 0, y > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Obtain

a) Joint moment generating function (mgf). (6 marks)

b) Marginal mgfs of X and Y. (3 marks)

c) Based on the joint mgf in (i) and marginal mgfs in (ii), can we say X and Y are stochastically independent? Why? (2 marks)

d) Use mgf to determine;

i) Expected values of X and Y. (4 marks)

ii) Variance of X and Y. (5 marks)

QUESTION THREE (20 MARKS)

Let X and Y be two independent random variables each uniformly distributed over the interval (0, 1). Let U and V be given in terms of X and Y as

$$U = X + Y \text{ and } V = Y - X$$

a) Determine the joint distribution of X and Y. (3 marks)

b) Jacobian of transformation from random variables. (4 marks)

c) The joint distribution of new variables U and V. (4 marks)

d) The marginal distributions of U and V. (9 marks)

QUESTION FOUR (20 MARKS)

a) Let X and Y be jointly distributed with p.d.f $f(x,y) = \begin{cases} x + y, 0 < x < 1 \\ \quad ,0 < y < 1 \\ 0, \text{ Otherwise} \end{cases}$

Obtain

i) $E(Y/X)$ (5 marks)

ii) $Var(Y/X)$ (6 marks)

b) Suppose that $X_1, X_2, \ldots, X_n$ be independent Bernoulli random variables with same p.m.f.

$$f(x) = \begin{cases} p^x (1-p)^{1-x}, & x = 0, 1 \\ 0, & \text{Otherwise} \end{cases}$$

Use moment generating function technique to obtain the distribution of $Y = \sum_{i=1}^{n} X_i$. (6 marks)

c) Suppose that the joint cumulative distribution function of X and Y is given by

$$F(x,y) = \begin{cases} \frac{1}{10}xy(x^2+y^2), & 0 \le x \le 2, \; 0 \le y \le 1 \\ 0, & \text{Otherwise} \end{cases}.$$

Find the joint probability distribution function $f(x,y)$ of X and Y. (3 marks)

QUESTION FIVE (20 MARKS)

a) Let X be a chi-square random variable with 10 degrees of freedom. Compute;

i) $P(X \ge 3.940)$. (2 marks)

ii) $P(X \le 2.558)$. (2 marks)

iii) $P(4.865 \le X \le 20.483)$. (3 marks)

b) Suppose T is a random variable having 16 degrees of freedom. Find;

i) $P(T > 1.071)$. (2 marks)

ii) $P(T \le 1.746)$. (2 marks)

iii) $P(0.865 \le T \le 2.921)$. (3 marks)

c) Suppose that X and Y are two independent and identically distributed random variables each having a probability distribution of the form

$$f(x) = \begin{cases} \frac{2x+4}{24}, & x = 1,2,3 \\ 0, & otherwise \end{cases}$$

Use moment generating function technique to find the probability distribution of $W = X + Y$.

(6 marks)