Kasarani Campus
Off Thika Road
Tel. 2042692 / 3
P. O. Box 49274, 00100
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
THIRD YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS)

Date: 6th December, 2023
Time: 11.30am –1.30pm

KMA 311 - PARTIAL DIFFERENTIAL EQUATIONS 1

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30MARKS)

a) Find the *p.d.e* by eliminating the arbitrary constants from,
$z = (x - a)^2 + (y - b)^2$ (4 Marks)

b) Classify the following 2nd order p.d.es as either parabolic, hyperbolic or elliptic.

i) $u_{xx} + u_{yy} = 0$ (1 Mark)

ii) $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = 0$ (1 Mark)

iii) $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ (1 Mark)

c) Solve the following equation subject to the given conditions
$\frac{\partial^2 u}{\partial x^2} = 24x^2(t-2)$, if at $x = 0$, $u = e^{2t}$ and $\frac{\partial u}{\partial x} = 4t$ (4 Marks)

d) Use the method of characteristics to find the general solution of
$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$, in which $y \geq 0$, Given that $u(x, 0) = f(x)$ (5 Marks)

e) Show that $z = ax + by - (a^2 + b^2)$ is a complete solution of
$z = px + qy - (p^2 + q^2)$
hence find the singular solution of the *p.d.e* (5 Marks)

f) Find the general and complete integral solution of
$a(p + q) = z$ where **a** is constant (4 Marks)

g) Solve $pq + qx = y$ using the Charpits method. (5 Marks)

QUESTION TWO (20MARKS)

a) Show that the conditions for exactness of the ordinary differential equation

$\mu(x,y)M(x,y)dx + \mu(x,y)N(x,y)dy = 0$ is a linear p.*d.e* or *order 1*.
Hence show how to find an integrating factor of $Mdx + Ndy = 0$

(8 Marks)

b) Using part (a), find an integrating factor for
$(2x^3y - y^2)dx - (2x^4 + xy)dy = 0$ (6 Marks)

c) Find the general solution of $(y+z)\frac{\partial z}{\partial x} + (z+x)\frac{\partial z}{\partial y} = x + y$

(6 Marks)

QUESTION THREE (20MARKS)

a) Solve the *p.d.e* $Pq = x^m y^n z^{2l}$ for a complete solution using the terms below as transformation.

$$Z = \frac{z^{1-l}}{1-l}, = \frac{x^{m+1}}{m+1}, Y = \frac{y^{n+1}}{n+1}$$ (6 Marks)

b) Show that by eliminating the arbitrary function f from $f(u,v) = 0$, where u and v are functions of x, y and z and $z = z(x,y)$, a *p.d.e* in the form

$P_p + Q_q = R$ is realised. (8 Marks)

c) Solve the equation $\frac{\partial^2 u}{\partial x \partial y} = \sin x \cos y$, subject to the boundary conditions that

at $y = \frac{\pi}{2}$, $\frac{\partial u}{\partial x} = 2x$ *and at* $x = \pi, u = 2\sin y$. (6 Marks)

QUESTION FOUR (20MARKS)

a) Derive Charpits system of differential equations for solving the *p.d.e* $f(x,y,z,p,q) = 0$

(9 Marks)

b) Solve the *p.d.e* $q = -xp + p^2$ using Charpits auxiliary system (7 Marks)

c) Find the *p.d.e* arising from (4 Marks)
$(x-a)^2 + (y-a)^2 + (z-b)^2 = 1$

QUESTION FIVE (20MARKS)

a) Using the transformation X= $\ln x$, $Y = \ln y$, find the **singular solution** of the equation

$z = x^2p^2 + y^2q^2$. ***(7 Marks)***

b) Solve the equation using the method of characteristics
$xu_y - yu_x = 0$ **given that** $u(0,y) = \cos y^2$ (7 Marks)

c) Find the complete solution of the P.D.E (6 Marks)