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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2016/2017 ACADEMIC YEAR
SECOND YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

Date: 9th August, 2016.
Time: 8.30am – 10.30am

KMA 204 - LINEAR ALGEBRA II

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) i) State the Cauchy-Scwartz Inequality.

(2 Marks)

ii) Verify the Cauchy-Scwartz Inequality for the vectors $u = (2, -3), v = (5, -7) in\ IR^2$.

(2 Marks)

b) The inner product on P_2 is defined as $< p, q >= a_o b_o + a_1 b_1 + a_2 b_2, where;$

$p(x) = a_o + a_1 x + a_2 x^2\ and\ q(x) = b_o + b_1 x + b_2 x^2$. Obtain $< p, q >$ for the vectors $p(x) = x^2$ and $q(x) = x - 3$.

(2 Marks)

c) For the rotation $\theta = \frac{3}{4}\pi\ find\ x'y'$ coordinate if the x y coordinates are (-2, 6); also find x y coordinates if $x'y'$ coordinates are (5, 2).

(4 Marks)

d) Find the cosine of the angle between $u = (-1, 5, 2) and\ v = (2, 4, -9)$ using Euclidean inner product in IR^3.

(5 Marks)

e) Given the matrix

$$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$

Find; i) Characteristic equation of A. (1Mark)

ii) Eigen values of A. (2 Marks)

iii) Basis for the eigen space of the matrix A. (4 Marks)

f) Let R^2 have the inner product $< x, y > = 3x_1y_1 + 2x_2y_2 \ where;$

$x = (-1, 2), y = (2, 5)$. Find

(i) $d(x, y)$

(2 Marks)

(ii) $\|x\|$

(2 Marks)

g) Let $x = \left(\frac{1}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right), and\ y = \left(\frac{2}{\sqrt{30}}, \frac{3}{\sqrt{30}}\right)$.

Show that $\{x, y\}$ is orthonormal if IR^2 has inner product defined in question *(f)* above but not orthonormal if IR^2 has Euclidean inner product.

(4 Marks)

<u>QUESTION TWO (20 MARKS)</u>

a) Define the terms;

i) Diagonalizable

(2 Marks)

ii) Orthonormal

(2 Marks)

b) Diagonalize the matrix;

$$A = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$$

(9 Marks)

c) Given that $S = \{v_1\ v_2\ v_3\} = \{(1, 2, 1,),\ (2, 9, 0,),\ (3, 3, 4,)\}$ is a basis for IR^3.

i) Find the coordinate vector and coordinate matrix of $v = (5, -1, 9,)$ w.r.t. S.

(4 Marks)

ii) Find the vector *v* in IR^3 whose coordinate vector w. r. t. S is $(v)_s = (-1, 3, 2)$.

(3 Marks)

<u>QUESTION THREE (20 MARKS)</u>

a) Given that $B = \{(1, 0), (0, 1)\}\ and\ B' = \{(1, 1,),\ (2,\ 1,)\}$ are basis for IR^2.

i) Find the transition matrix P from $B'to\ B$.

(9 Marks)

ii) If $[W]_{B'} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}, find\ \ [W]_B$

(2 Marks)

b) State the Cayley-Hamilton theorem; hence use it to find the inverse of the matrix;

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(9 Marks)

QUESTION FOUR (20 MARKS)

a) Transform the set $S = \{1, x, x^2\}$ which is a basis for P_2 into an orthonormal basis w. r. t. the integral inner product on P_2 defined as;

$$< p, q > = \int_{-1}^{1} p(x)q\,(x)dx.$$

(10 Marks)

b) Find a matrix P that diagonalizes

$$A = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

(10 Marks)

QUESTION FIVE (20 MARKS)

Identify the curve which is represented by the following quadratic equation by first putting it into standard conic form $x^2 + 2xy + y^2 - x + y = 0.$