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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2020/2021 ACADEMIC YEAR
FIRST YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS)

Date: 17th December, 2020
Time: 11.30am – 1.30pm

KMA 2102 - DIFFERENTIAL CALCULUS 1

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) Explain each of the following terms as used in calculus
- i) Differentiation (1 Mark)
 - ii) Function (1 Mark)
 - iii) Limit point of a function (1 Mark)
- b) Find the exact value of $\lim_{x \rightarrow \infty} \sqrt{x^3 - 4} - \sqrt{x^3 + x}$ (4 Marks)
- c) Find $\frac{dy}{dx}$ if $y = \sec x$ (4 Marks)
- d) Evaluate $\lim_{x \rightarrow 0} \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{x} \right)$ (3 Marks)
- e) Evaluate the derivative of $x + y^2 = e^{xy^3}$ (4 Marks)
- f) From the definition of derivatives, find the derivative of $f(x) = 3 - 2x - x^2$ (3 Marks)
- g) Let $x = t^3$ and $y = t^2 - t$. Obtain the first derivative of y with respect to x (3 Marks)
- h) Find the derivatives of the following
- i) $y = \ln(x^2 + 1)$ (2 Marks)
 - ii) $y = \frac{1 - \cos x}{1 + \cos x}$ (2 Marks)
 - iii) $y = x^3(2x - x^4)$ (2 Marks)

QUESTION TWO (20 MARKS)

- a) Sand is falling in a conical pipe at a rate of $100m^3 / \text{min}$. Find the rate of change of the height when the height is $10m$ assuming that the coasance of the sand is such that the sand is equal to the radius. (12 Marks)
- b) Evaluate $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 4}$ (4 Marks)
- c) Find the derivative of the following function $x^2 + y^2 = 1$ (4 Marks)

QUESTION THREE (20 MARKS)

- a) A drug is injected into the bloodstream of a patient through the right arm. The concentration of the drug in the bloodstream of the left arm t hours after the injection is approximated by $c(t) = \frac{0.14t}{t^2 + 1}$ $0 \leq t \leq 24$. Determine $c(3)$ and interpret your results (3 Marks)
- b) Find the derivative of each of the following equations
- i) $y = \frac{3 - 2x^2}{3 + 2x}$ (5 Marks)
- ii) $x = y\sqrt{1 - y^3}$ (5 Marks)
- c) The position function of a particle is given by $s(t) = 3t^2 - t^3, t \geq 0$
- i) When does the particle reach a velocity of $0m/s$? Explain the significance of this value of t (4 Marks)
- ii) When does the particle have acceleration $0m/s^2$? (3 Marks)

QUESTION FOUR (20 MARKS)

- a) Using the function $f(x) = 25 - 5x^2$ find
- i) Its derivative from the first principle (7 Marks)
- ii) The rate of change of $f(x)$ at $x = 1$ (1 Mark)
- b) Differentiate the following functions with respect to x
- i) $y = \left(\frac{x}{1+x}\right)^5$ (4 Marks)
- ii) $y = \cos^2 3x$ (4 Marks)
- c) Find the equation of the normal to the curve $y = x^2 + 3x$ at the point where $x = 1$ (4 Marks)

QUESTION FIVE (20 MARKS)

- a) Find the equation of the tangent line to the graph of $f(x) = \sqrt{x^2 + 3}$ at the point $(-1, 2)$ (6 Marks)
- b) Find all critical points of $f(x) = x^4 - 8x^2$. (5 Marks)
- c) Evaluate $\lim_{x \rightarrow -3} \frac{x^2 - 9}{x + 3}$ (4 Marks)
- d) Using the definition of a derivative, find the derivative of $f(x) = x^2$. (5 Marks)