Kasarani Campus
Off Thika Road
Tel. 2042692 / 3
P. O. Box 49274, 00100
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2024/2025ACADEMIC YEAR
THIRD YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF EDUCATION
(ARTS)

Date: 8th April, 2024
Time: 11.30am –1.30pm

KMA 2301 - REAL ANALYSIS

INSTRUCTIONS TO CANDIDATES
ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) Give an example of a set which has got an infimum, a minimum and a supremum but does not have a maximum element. (2 Marks)

b) State whether the following sets are open, closed or neither
   i) $S = \{1,2,3\}$ (1 Marks)
   ii) $S = (-\infty, 1)$ (1 Mark)
   iii) $\mathbb{R}$ (1 Mark)

c) Let (X, ρ) be a metric space. Define the following terms:
   i) Neighborhood of a point. (1 Mark)
   ii) Interior point. (2 Marks)
   iii) Limit point. (2 Marks)
   iv) Isolated point . (1 Mark)

d) Given a set $S = \{1,2\} \cup (3,11)$. Find the following:
   i) S° (1 Mark)
   ii) $\bar{S}$ (1 Mark)
   iii) Is S (1 Mark)
   iv) ∂S (1 Mark)

e) Prove that the set of irrational numbers is uncountable. (3 Marks)

f) Prove that an empty set is open. (4 Marks)

g) Show that the infinite series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent. (2 Marks)

h) Prove using mathematical induction that for all $n \geq 1$, $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ (3 Marks)

   i) Let X be a non-void set and $\rho : \boldsymbol{X} \times \boldsymbol{X} \to \boldsymbol{X}$ be defined by $\rho(x,y) = |x - y|$ Show that (X, ρ) is a metric space. (3 Marks)

QUESTION TWO (20 MARKS)

a) Prove that the intersection of an arbitrary family of closed sets is closed. (6 Marks)

b) Prove that the set of rational numbers is countable. (6 Marks)

c) i) Show that the infinite set $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots\ldots\ldots\ldots\ldots\ldots. is\ bounded$ (2 Marks)

ii) Determine the supremum and the infimum of the set in question c i) above. (2 Marks)

d) Use ratio test to test the convergence of the series $\sum_{n=1}^{\infty} \frac{n!(n+1)!}{(3n)!}$. (4 Marks)

QUESTION THREE (20 MARKS)

a) Prove that every convergent sequence has a unique limit. (6 Marks)

b) Give a counter example to show that if sequence is bounded it is not necessarily convergent. (4 Marks)

c) For any four numbers a, b, c, d assume that a<b and c<d and prove that a+c < b+d. (5 Marks)

d) Prove that for positive numbers x and y , x<y then if and only if $x^2 < y^2$. (5 Marks)

QUESTION FOUR (20 MARKS)

a) Let X be a non-void set and $\rho : X \times X \to X$ be defined by $\rho(x,y) = \begin{cases} 1 \text{ if } x \neq y \\ 0 \text{ if } x = y. \end{cases}$ Show that (X, ρ) is a metric space. (6 Marks)

b) Prove that is a set S has a mimima in ℝ, then this minima is unique. (4 Marks)

c) Given the set $S = [-\infty, 4) \cup \{5, 9\} \cup [6,7)$, find:

i) A° (1 Mark)

ii) ∂A (1 Mark)

iii) $(A^c)^{\circ}$ (2 Marks)

iv) $\overline{A}$ (1 Mark)

d) State and prove the principle of Archimedean. (5 marks)

QUESTION FIVE(20 MARKS)

a) State Cauchy root test and hence use it to test the convergence of

$$\sum_{n=0}^{\infty} \left(\frac{5n-3n^3}{7n^3+2}\right)^n$$ (5 marks)

b) Use integral test to test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n-1}}$ (5 Marks)

c) i) State the completeness axiom. (1 Mark)

ii) Illustrate using an example that the set of rational numbers doesn't satisfy the completeness axiom. (4 Marks)

d) Prove that the intersection of finite number of open sets is open. (5 marks)