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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2016/2017 ACADEMIC YEAR
SECOND YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

Date: 10th August, 2016.
Time: 8.30am – 10.30am

KMA 207 - THEORY OF ESTIMATION

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Suppose that $x_1, x_2, \cdots, x_n$ form a random sample of size n from a population with density function;

$$P(X=x) = \begin{cases} \binom{n}{x}\theta^x(1-\theta)^{n-x}, x = 0,1,2,\cdots,n \\ 0\,, elsewhere \end{cases}$$

Show that $x/_n$ is an unbiased estimator of θ.

(6 Marks)

b) State and explain the steps involved in statistical analysis.

(4 Marks)

c) Suppose that $x_1, x_2, \cdots, x_n$ form a random sample of size n from a normal population with mean μ and variance σ^2. Let $\bar{x}_n = \frac{1}{n}\sum_{i=1}^{n} x_i$ be the sample mean, show that $\bar{x}_n$ is a consistent estimator of μ.

(6 Marks)

d) Consider a poisson distribution given by $f(x,\theta) = \frac{e^{-\theta}\theta^x}{x!}$, $x = 0, 1, 2, \cdots, n$. Show that $T = \sum_{i=1}^{n} x_i$ is a sufficient statistic for parameter θ.

(5 Marks)

e) 12 randomly selected mature citrus trees of one variety have mean height of 13.8 feet with standard deviation of 1.2 feet and 15 randomly selected mature citrus trees of another variety have a mean height of 12.9 feet with standard deviation of 1.5 feet. Assuming that the random samples were selected from normal populations with equal variances, construct a 95% confidence interval for the difference in the true average heights of the 2 kinds of citrus trees.

(4 Marks)

f) Let $x_1, x_2, \cdots, x_n$ be a random sample from a Bernoulli population with density function $f(x,p) = \begin{cases} p^x(1-p)^{1-x}, x = 0,1 \\ 0, elsewhere \end{cases}$. If $d(\underline{x})$ is an unbiased estimator of p, find the Crammer-Rao lower bound for the unbiased estimator of p.

(5 Marks)

QUESTION TWO (20 MARKS)

a) A paint manufacturer wants to determine the average drying time of a new interior wall paint. If 9 test areas of equal size, he obtained a mean drying time of 88.4 minutes and a standard deviation of 9.6 minutes. Construct a 95% CI for the true mean μ.

(3 Marks)

b) Find the maximum likelihood estimator of unknown parameter p of Bernoulli distribution.

(7 Marks)

c) $f(x,\theta) = \begin{cases} (\theta_1 + 1)x^{\theta_2}, 0 < x < 1 \\ 0, elsewhere \end{cases}$

Find the estimator of θ_1 and θ_2 using the method of moments.

(10 Marks)

QUESTION THREE (20 MARKS)

a) Let $x_1, x_2, \cdots, x_n$ denote a random sample from normal distribution with both unknown mean(μ) and variance(σ^2). Show that the statistic $T_1 = \bar{x}$ and $T_2 = \sum_{i=1}^{n}(x_i - \bar{x})^2$ are jointly sufficient

(10 Marks)

b) Let $S_{n-1}^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1}$ be the sample variance of a random sample $x_1, x_2, \cdots, x_n$ from an infinite population with unknown variance σ^2. Prove that S_{n-1}^2 is an unbiased estimator of σ^2.

(10 Marks)

QUESTION FOUR (20 MARKS)

a) Given that $y_i = \alpha + \beta x_i + e_i$ where e_i is the random error term and that $E(y_i) = \alpha + \beta x_i$, obtain the best estimates of α and β.

(11 Marks)

b) Define loss function hence or otherwise state the four possible loss functions.

(5 Marks)

c) In Bayes's estimation, what is prior distribution, hence give the notations used in this method of estimation.

(4 Marks)

QUESTION FIVE (20 MARKS)

a) Define uniformly minimum variance unbiased estimator (UMVUE).

(3 Marks)

b) $f(x) = \begin{cases} \theta e^{-\theta x}, 0 \le x \le \infty \\ 0, elsewhere \end{cases}$.

i) Find the Crammer-Rao lower variance of the unbiased estimator of $g(\theta) = \theta$

(8 Marks)

ii) Show that $\bar{x}$ is the MVBUE of $\frac{1}{\theta}$

(9 Marks)