Kasarani Campus
Off Thika Road
Tel. 2042692 / 3
P. O. Box 49274, 00100
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2016/2017 ACADEMIC YEAR
FIRST YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

Date: 12th August, 2016.
Time: 11.00am – 1.00pm

KMA 103 – LINEAR ALGEBRA I

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Solve the following system of linear equations using Gaussian elimination method;

$$2x + y + z = 1$$
$$-x + 2y - 3z = 3$$
$$x + 3y - 2z = 4$$

(5 Marks)

b) i) Define linear dependence and independence of vectors

(2 Marks)

ii) Determine whether the following polynomials are linearly independent or dependent.

$$p(x) = 2 + x - x^2, q(x) = x - 2x^2, h(x) = 2 + 3x - x^2$$

(6 Marks)

c) Which of the following are subspaces of R^3?

i) All vectors of the form (a, 0, 0)

ii) All vectors of the form (a, b, c) where $b = a + c$.

iii) All vectors of the form (a, 1, 1)

(3 Marks)

d) Let $T: R^4 \rightarrow R^3$ be a linear transformation defined by;

$$T(X) = \begin{bmatrix} 4 & 1 & -1 & -3 \\ 2 & 1 & 1 & -4 \\ 6 & 0 & -9 & 9 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

Find;

i) Nullity of T.

ii) Rank of T.

(5 Marks)

e) Find a basis for the column space of $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 3 & 2 & 5 & 1 \\ 0 & 4 & 4 & -4 \end{bmatrix}$

(4 Marks)

f) Find the values of k for which the matrix $T = \begin{bmatrix} k-3 & 4 \\ k & k+2 \end{bmatrix}$ has no inverse.

(5 Marks)

QUESTION TWO (20 MARKS)

a) Find the inverse of the following matrix using row reduction method;

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

(8 Marks)

b) Prove thatif A is an $n \times n$ matrix which is invertible, then for any vector $\bar{b}$in R^n the linear system $A\underline{x} = \underline{b}$ has a unique solution $A^{-1}\, b$. Hence find a unique solution for the system

$$3x + 2y + 3z = 1$$
$$2x - 2y + 4z = 6$$
$$4x + 5y - z = -2$$

(9 Marks)

c) Let $A = \begin{bmatrix} 1 & -3 & -2 \\ -5 & 9 & 1 \end{bmatrix}$, and let $u = \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix}$. Determine if u belongs to the null space of A.

(3 Marks)

QUESTION THREE (20 MARKS)

a) Find the basis and dimension of the solution space for the equations;

$$2x_1 + 2x_2 - x_3 + x_5 = 0$$
$$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$$
$$x_1 + x_2 - 2x_3 - x_5 = 0$$
$$x_3 + x_4 + x_5 = 0$$

Also find the rank of the coefficient matrix.

(8 Marks)

b) Find the rank and nullity of the linear transformation whose associated matrix is $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \end{bmatrix}$

(5 Marks)

c) Determine if $S = \{(2,2), (-3,3)\}$ is linearly independent.

(7 Marks)

QUESTION FOUR (20 MARKS)

a) Find the inverse of the following matrix by first getting its adjoint

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \\ 1 & 0 & 3 \end{bmatrix}$$

(8 Marks)

b) Determine the value of $'a'$ so that;

$$x_1 - 3x_3 = -3$$
$$2x_1 + ax_2 - x_3 = -2$$
$$x_1 + 2x_2 + ax_3 = 1$$

has i) No solution

ii) Unique solution

iii) Many solution.

(8 Marks)

c) Show that $(2,-1)$ is the set generated by $S = \{(3,1), (2,2)\}$

(4 Marks)

QUESTION FIVE (20 MARKS)

a) Express the vector $\underline{u} = (0,2,1)$ as a linear combination of the vectors $\underline{v_1} = (-1,1,0), \underline{v_2} = (2,0,1), \underline{v_3} = (1,1,1)$.

(8 Marks)

b) Show that $S = \{(1,2,1), (2,9,0), (3,3,4)\}$ is a basis for R^3.

(7 Marks)

c) Find the dimension of the row space of the matrix $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 5 \\ -1 & 2 & 2 \end{bmatrix}$

(5 Marks)