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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
FIRST YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF BUSINESS AND INFORMATION
TECHNOLOGY

Date: 7th December, 2023
Time: 8.30am – 10.30am

KMA 2102 - CALCULUS FOR BUSINESS INFORMATION

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) Evaluate the limits
- $\lim_{x \rightarrow 1} \frac{x-2}{x^2+x-6}$ (2 marks)
 - $\lim_{x \rightarrow \infty} \frac{2x^4 - x^2 + 8x}{-5x^4 + 7}$ (2 marks)
 - $\lim_{x \rightarrow 0} \frac{\log(1+x)}{x}$ (2 marks)
- b) Differentiate $f(x)$ from first principles given $f(x) = -2x^2 + 3$ (3 marks)
- c) A particle moves along a straight line so that its displacement, S metres, from a fixed point in it, after t seconds is given by $s(t) = 6t^2 - t^3$.
- Find the time after the start when the velocity is zero. (2 marks)
 - At what time after the start is the acceleration zero? (2 marks)
- d) What value must be assigned to k so that the following function is continuous at $x = 4$
- $$f(x) = \begin{cases} \frac{x^2-25}{x-5}, & x \neq 5 \\ k, & x = 5. \end{cases}$$
- (3 marks)
- e) Given $f(x) = 3x - 2$ and $h(x) = x^3$, find each of the following:
- $(f \circ h)(x)$ (2 marks)
 - $(h \circ f)(x)$ (2 marks)
- f) Given that $f(x) = \frac{x+4}{2x-5}$, find $f^{-1}(x)$, the inverse of $f(x)$. (2 marks)
- g) Differentiate the following functions:
- $y = x^2 - \frac{1}{2x^3}$. (3 marks)
 - $y = \tan(3x^2 + 2x + 5)$. (3 marks)
 - $y = e^x(2x^2 + 1)$ (2 marks)

QUESTION TWO (20 MARKS)

- a) Find $\frac{dy}{dx}$ for the following
- $y = \ln(x^2 + 3x + 1)$ (3 marks)
 - $y = \frac{3x^4}{(x-1)^4}$ (3 marks)
 - $y = \frac{1+\cos x}{1-\cos x}$ (3 marks)
- b) Find domain and range for each of the following functions.
- $f(x) = x^2 + 4x - 5$ (2 marks)
 - $f(x) = \sqrt{x+2} - 4$ (2 marks)
 - $f(x) = \frac{5}{x-3}$ (2 marks)
- c) Consider the curve $y^2 + xy - x^2 = 7$. Find the equation of the tangent line to the curve at the point $(2, 3)$. (3 marks)

QUESTION THREE (20 MARKS)

- a) Differentiate the following function implicitly
- $y^2 + x^3 - y^3 + 6 = 3y$ (3 marks)
 - $y^2 + x^3 - xy + \cos y = 0$ (3 marks)
- b) Differentiate the following functions:
- $f(x) = e^{x^2}(2x^2 + 3x)$ (2 marks)
 - $y = \ln(x^2 + 2)$ (2 marks)
 - $f(x) = e^{3x} \sin(2x + 1)$ (2 marks)
 - $y = \left(\frac{x}{3x+2}\right)^5$ (2 marks)
 - $y = 5x^3 \sin x$ (2 marks)
 - $f(x) = \frac{\cos x}{4x^2}$ (2 marks)
 - $y = \cos^4(7x^2 + 1)$ (2 marks)

QUESTION FOUR (20 MARKS)

- a) Find the equation of the line tangent to the given curve $x = 2t - 1, y = 4t^2 - 2t$ at the point $t=1$. (4 marks)
- b) Show that $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$ (5 marks)
- c) A spherical balloon is being filled with air at the constant rate of $2 \text{ cm}^3/\text{sec}$. How fast is the radius increasing when the radius is 3 cm? (5 marks)

- d) Find the derivatives of the following functions
- i) $y = \frac{\sin x}{x}$ (2 marks)
- ii) $y = \sin x \cos x$ (2 marks)
- iii) $y = 5e^x + \cos x$ (2 marks)

QUESTION FIVE (20 MARKS)

- a) Given the following function $f(x) = -x^3 + x^2 + 8 - 12 - 4x^3$.
- i) Find the x-intercepts of the graph f. (3 marks)
- ii) Calculate the turning points of the given function (3 Marks)
- iii) Sketch the graph (2 Marks)
- b) Show that $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3} = -\frac{1}{6}$. (3 Marks)
- c) A soccer ball is kicked into the air so that the path of its flight can be modeled by the function $x(t) = -4.9 t^2 + 9.8t + 5$ where t is in seconds and x is meters above the ground.
- i) At what time will the ball land? (2 marks)
- ii) How many meters above the ground was the ball kicked? (2 marks)
- iii) Find the maximum height the ball will reach and at what time will this occur? (3 marks)
- iv) What is the acceleration of the ball at $t = 3$ s? (2 Marks)