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RIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR

END SEMESTER EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

Date: 17th AUGUST 2023

Time: 8:30am – 10:30am

KMA 203: PROBABILITY AND STATISTICS II

INSTRUCTIONS TO CANDIDATES:

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Consider the following joint distribution function defined as;

$$f(x,y)=\begin{cases}\frac{1}{8}(6-x-y); & 0<x<2\,;\ 2<y<4\\ 0 & ;\ elsewhere\end{cases}$$

i. Find $\Pr(X+Y>1)$ (3 Marks)

ii. Find the marginal distribution of x (2 Marks)

b) Let $x_1, x_2, \cdots x_n$ be a random sample of size n from a population with mean μ and variance σ^2. Let $\bar{x}_n$ be the sample mean. Prove that;

$$\operatorname{var}(\bar{x}_n)=\frac{\sigma^2}{n}$$

(3 Marks)

c) Determine whether X and Y are independent given that their joint probability density function is as follows (4 Marks)

$$f(x,y)=\begin{cases}6xy^2, & 0\le x\le 1\,;\ 0\le y\le 1\\ 0\ , & elsewhere\end{cases}$$

d) The bivariate density function of random variables X and Y is given by

$$f(x,y)=\begin{cases}\frac{3}{5}x(y+x), & 0<x<1\,;0<y<2\\ 0, & elsewhere\end{cases}$$

Find $\operatorname{var}(XY)$ (6 Marks)

e) Prove that $E(E(X|Y))=E(X)$ (3 Marks)

f) The joint discrete bivariate distribution of X_1 and X_2 is given by

$$f(x_1 x_2)=\begin{cases} p^{x_1+x_2}(1-p)^{2-x_1-x_2}, & x_1=0,1;\ x_2=0,1 \\ 0\ , & elsewhere \end{cases}$$

Find;

i. $E(X_1)$ (5 Marks)

ii. $E(X_1+X_2)$ (4 Marks)

QUESTION TWO (20 MARKS)

a) The random variables x and y have the following joint distribution function;

$$f(x,y)=\begin{cases} ke^{-2x-5y}, x>0;\ y>0 \\ 0,\ elsewhere \end{cases}$$

i. Find the value of k (3 Marks)

ii. Hence find the joint moment generating function of x and y $M_{xy}(t_1 t_2)$ (7 Marks)

b) Let the joint probability distribution of X and Y have a joint probability distribution function given as

$$f(x,y)=\begin{cases} k(1-x),\ 0<x<y<1 \\ 0,\ elsewhere \end{cases}$$

i. Determine the value of k (2 Marks)

ii. Compute;

i) $pr(X+Y\leq 0.7)$ (4 Marks)

ii) $pr(Y|X=0.5)$ (4 Marks)

QUESTION THREE (20 MARKS)

a) In an experiment to test two procedures, the following information was obtained from the runs

$$n_2=9 \quad \bar{x}_2=35.22\sec s \quad \sum_{i=1}^{9}(x_{2i}-\bar{x})^2=160.22$$

$$n_1=9 \quad \bar{x}_1=31.56\sec s \quad \sum_{i=1}^{9}(x_{1i}-\bar{x})^2=195.56$$

Test hypothesis that the two populations have the same mean. Take $\alpha=0.01$ level of significance. (3 Marks)

b) Let $x_1, x_2, \cdots x_n$ be a random sample from a population with mean μ and variance σ^2. Define $s_n^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$. Show that $E(s_n^2) = \sigma^2$ (6 Marks)

c) Suppose that X and Y are two discrete random variables with a probability function given by

$$f(x,y) = \begin{cases} \frac{1}{54}(x+y), & x=1,2,3;\ y=1,2,3,4 \\ 0, & elsewhere \end{cases}$$

i. Determine the marginal probability functions for X and Y respectively. (4 Marks)

ii. Find $Var(Y|X=2)$ (7 Marks)

QUESTION FOUR (20 MARKS)

a) Suppose that the joint probability distribution function of X_1 and X_2 is

$$f(x_1, x_2) = \begin{cases} k(x_1 + x_2), & x_1 = 0,1,2;\ x_2 = 0,1,2,3,4 \\ 0, & elsewhere \end{cases}$$

Find;

i. The value of k (2 Marks)

ii. $P(X_2 = 3 | X_1 = 1)$ (5 Marks)

b) The joint probability distribution function of X_1 and X_2 is given by

$$f(x_1 x_2) = \begin{cases} 6x_1 x_2 (2 - x_1 - x_2), & 0 \le x_1 \le 1; 0 \le x_2 \le 1 \\ 0, & elsewhere \end{cases}$$

Find;

i. $\text{var}(X_1)$ (4 Marks)

ii. $\text{cov}(X_1 X_2)$ and interpret the result (9 Marks)

QUESTION FIVE (20 MARKS)

a) Suppose x_1, x_2, x_3 is a random sample of size $n=3$ from a normal distribution with unknown mean μ and variance of 54. Define $\bar{x}_3 = \frac{1}{3} \sum_{i=1}^{3} x_i$ to be the sample mean. Calculate $p(|\bar{x}_3 - \mu| < 3)$ (7 Marks)

b) Let $x_1, x_2, \cdots x_n$ be a random sample of size n from a standard normal distribution. Prove that $\bar{x}$ and $\sum_{i=1}^{n} (x_i - \bar{x})^2$ are independent. (13 Marks)