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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2024/2025ACADEMIC YEAR
FIRST YEAR, FIRSTSEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS AND COMPUTER SCIENCE)

Date: 15th April, 2024
Time: 8.30am –10.30am

KMA 104 - CALCULUS 1

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) Evaluate the limits of the following functions

i) $\lim_{x\to -3}\left(\frac{x^2-9}{x+3}\right)$ (2 marks)

ii) $\lim_{x\to \infty}\left(\frac{16x^3-4x^2+2x+10}{2x^3-5x+6}\right)$ (2 marks)

b) Given the functions $f(x)=x^2$ and $g(x)=8x+1$, find:

i) $f\circ g$ (2 marks)

ii) $g\circ f$ (1 mark)

c) Use first principle of differentiation to find $\frac{dy}{dx}$ given that $y(x)=\frac{1}{3-5x}$

(3 marks)

d) Find the derivative of the following functions;

i) $y = x^{tan(x^2)}$ (2 marks)

ii) $x = 3t^2 - 4t + 6$ and $y = 16 - t^4$ (2 marks)

iii) $y = \frac{sinx}{4x^2}$ (2 marks)

iv) $y = \ln(x^2 + 2)$ (2 marks)

v) $y = e^{3x}\sin(2x + 1)$ (2 marks)

vi) $y = \left(\frac{x}{3x+2}\right)^5$ (2 marks)

e) If $y = \sin^{-1}(3x - 4x^3)$, show that $\sqrt{1 - x^2}\frac{dy}{dx} = 3$ (3 marks)

f) A ball is thrown vertically upwards so that its height is S meters after t seconds is given by $S = \frac{1}{27}t^2 + 4\sqrt{t}$. find its:

i) Velocity at any time t (1 mark)

ii) Acceleration when $t = 1$ (2 marks)

iii) Maximum height reached (2 marks)

QUESTION TWO (20 MARKS)

a) Use chain rule to differentiate the following function $y = \left(\frac{1+2x}{1+x}\right)^2$ (5 marks)

b) Determine the values of constants A and B so that the following function is continuous everywhere on the real number line.

$$f(x) = \begin{cases} 2, & if\ x < 1 \\ Ax + B, & if\ 1 \le x \le 2 \\ 6, & if\ x > 2 \end{cases}$$ (5 marks)

c) Find the derivative of the function represented by $x = \frac{2t}{1+3t}$ and $y = t^3 - 4t + 8$

(5 marks)

d) Determine the co-ordinates and the nature of the turning points on the curve defined by $y = 2x^3 + 9x^2 - 60x$ (5 marks)

QUESTION THREE (20 MARKS)

a) If y is a differentiable function of x, find the derivative of y with respect to x given that

i) $y = x^3 e^{x^3 + 4\sqrt{x}} - cos(ln2x)$ (5 marks)

ii) $x = 1 + 3t^2$ and $y = \frac{2-t^2}{1+3t^2}$ (5 marks)

b) Find $\frac{dy}{dx}$ if $y = sin(\sqrt{x})$

(4 marks)

c) In marketing a certain item, a business has discovered that the demand for the item is represented by $p(x) = \frac{50}{\sqrt{x}}$. The cost of producing x items is given by $c(x) = \frac{1}{2}x + 500$. Find the price per unit that will yield maximum profit.

(6 marks)

QUESTION FOUR (20 MARKS)

a) Find the derivative of the following function
$y = 3x^4 + 2x^3 + x + 7$ (3 marks)

b) Find the equation of the normal to the curve $x^2 + 2xy + 3y^2 = 1$ at point $(2,1)$
(5 marks)

c) A rectangular box is to be made from a piece of cardboard measuring 24 inches long by 9 inches wide by cutting out identical squares from the four corners and turning up the sides. Find the dimensions of the box that will maximize the volume. What is the maximum volume? (7 marks)

d) Evaluate $\lim_{x \to 8} \left(\frac{|x-8|}{(x-8)} \right)$

(5 marks)

QUESTION FIVE(20 MARKS)

a) Find the derivative of the following function with to x
$y = a^x$ (5 marks)

b) Classify the extreme points of $y(x) = 2x^3 + 3x^2 - 12x$
(4 marks)

c) A manufacturer wants to design an open box having a square base and surface area of $108m^2$. Find the dimensions of the box that will give maximum volume.
(7 marks)

d) Find $\frac{dy}{dx}$ given the following function $y = \frac{2-x^2}{1+3x^2}$
(4 marks)