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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
FIRST YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF BUSINESS INFORMATION TECHNOLOGY
KMA 2114 - MATHEMATICAL LOGIC

Date: 14th August, 2023
Time: 8.30am – 10.30am

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Using logical equivalence laws show that $(p \wedge (\sim p \wedge q)) \Leftrightarrow \sim p \wedge \sim q$ (4 Marks)

b) Prove that $\sqrt{3}$ is irrational by contradiction (5 Marks)

c) List the members of these sets

i) ¿ ¿ and x is a sexy prime}

ii) {x| x is a real number such that $x^2 = 1$ (4 Marks)

d) Write the inverse, converse and contrapositive of the given statement "If Kenya can qualify for AFCON, then Kenya can finish third in the competition (4 Marks)

e) Suppose that A is the set of sophomores at your school and B is the set of students in discrete mathematics at your school. Express each of these sets in terms of A and B.

i) the set of sophomores taking discrete mathematics in your school (1 Mark)

ii) the set of sophomores at your school who are not taking discrete mathematics (1 Mark)

iii) the set of students at your school who either are sophomores or are taking discrete mathematics (1 Mark)

iv) the set of students at your school who either are not sophomores or are not taking discrete mathematics (1 Marks)

f) Name the following laws of arithmetic

i) $(x+y)+z = x+(y+z)$ (1 Mark)

ii) $xy = yx$ (1 Mark)

iii) $x(y+z) = xy+xz$ (1 Mark)

g) Using a Venn diagram illustrate that $A \cup (A \cap B) = A$ (3 Marks)

QUESTION TWO (20 MARKS)

a) Distinguish between a tautology and a contradiction. (2 Marks)

b) Test the validity of the following argument.

$p \vee q \; p \longrightarrow r \; q \longrightarrow r \qquad \therefore r$

(6 Marks)

c) Given that $f(x) = 2x$, $g(x) = x^2$ and $h(x) = x+1$, find:

i) $f \circ (g \circ h)$

ii) $g \circ (f \circ h)$ (4 Marks)

d) A survey on a sample of 25 new cars being sold at a local auto dealer was conducted to see which of three popular options, air-conditioning *(A)*, radio *(R)*, and power windows *(W)*, were already installed. The survey found: 15 had air-conditioning *(A)*, 5 had *A* and *W*, 12 had radio *(R)*, 9 had *A* and *R*, 3 had all three options. 11 had power windows *(W)*, 4 had *R* and *W*. Represent this information in a well labelled Venn diagram and hence find the number of cars that had:

i) Only *W*
ii) *R* and *W* but not *A*
iii) Only one of the options
iv) None of the options (8 Marks)

QUESTION THREE (20 MARKS)

a) Write the converse, inverse and contrapositive of the following statement "If someone has read 'No longer at ease,' then he remembers the character of Obi" (6 Marks)

b) Let $f: R \to R$ be defined by $f(x) = 2x - 3$. Find f^{-1} (4 Marks)

c) Disapprove by counter example that "for all prime numbers $p, 2p+1$ is prime (3 Marks)

d) Prove by mathematical induction that $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$ (7 Marks)

QUESTION FOUR (20 MARKS)

a) Using a Venn diagram to show that $\overline{A \cup B} = \overline{A} \cap \overline{B}$, if *A* and *B* are sets. (4 Marks)

b) Use mathematical induction to prove that $1 + 2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 1$ (4 Marks)

c) Let $f: R \to R$ and $g: R \to R$ be defined by $f(x) = 7x + 1$ and $g(x) = -2$. Find the formula for the composition functions *gof*, *fog* and *fof* (6 Marks)

d) Prove that $\sqrt{p}$ is irrational by contradiction. (6 Marks)

QUESTION FIVE (20 MARKS)

a) Let *p* and *q* be the propositions; *p*: I played in AFCON for the first time.
q: I won the AFCON.
Express proposition $\neg p \vee (p \wedge q)$ as an English sentence. (3 Marks)

b) Use mathematical induction to prove that $12^n - 1$ is divisible by $11, \forall n \in N.$ (7 Marks)

c) Find the number of integers between 1 and 100 inclusively that are divisible by either 5 or 7 (5 Marks)

d) Let p and q denote: "I bet for the first time today", and "I win the 1 million jackpot" respectively. State the verbal translation of each of the following;

i) $p \wedge q$
ii) $\neg p \vee q$
iii) $\neg p \wedge \neg q$
iv) $\neg(p \vee \neg q)$
v) $\neg(\neg p \vee \neg q)$ (5 Marks)