Kasarani Campus
Off Thika Road
Tel. 2042692 / 3
P. O. Box 49274, 00100
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2020/2021 ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF EDUCATION ARTS

Date: 15th December, 2020
Time: 8.30am – 10.30am

KMA 2204 - LINEAR ALGEBRA 11

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) i) Compute $< u, v >$ using the inner product $\mathbb{R}^2$ defined by $< u, v >= 3u_1v_1 + 2u_2v_2$ where $u = (2,3), v = (-4,5)$. (2 Marks)

ii) Use the inner product defined by
$$< f, g >= \int_0^1 f(x)g(x)dx$$ to compute the $< f, g >$ for the vectors of $f(x) = x$ and $g(x) = e^x$ in $C[0,1]$. (4 Marks)

b) Let P_2 have the inner product defined by
$< p, q >= a_o\, b_o + a_1\, b_1 + a_2 b_2$, where
$p(x) = a_o + a_1 x + a_2\, x^2$ and $q(x) = b_o + b_1 x + b_2\, x^2$.
Find $\|p\|$ where $p = -1 + x^2 + 2x$. (4 Marks)

c) Let $A = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$. Compute A^5 (5 Marks)

d) Find the coordinate vector and coordinate matrix of the vector $v = (2, -1, 3)$ relative to the basis $S = \{(1,0,0), (2,2,0), (3,3,3)\}$. (5 Marks)

e) Given the matrix $A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$

i) Find the characteristic equation of A. (2 Marks)

ii) Find the eigenvalues of matrix A. (2 Marks)

iii) Inverse of matrix A. (3 Marks)

f) Find a unit vector orthogonal to both $v_1 = (1,1,2)$ and $v_2 = (0,1,3)$ (3 Marks)

QUESTION TWO (20 MARKS)

a) Given $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$. Find a matrix B such that $B^2 = A$. (7 Marks)

b) Consider the bases $B = \{u_1, u_2, u_3\} = \{(1,0,0), (0,1,0), (0,0,1)\}$ and $B' = \{v_1, v_2, v_3\} = \{(1,0,1), (0,-1,2), (2,3,-5)\}$ for $\mathbb{R}^3$.

i) Find the Transition matrix from B to B'. (7 Marks)

ii) Compute the coordinate matrix $[x]_B$, where $x = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$. (3 Marks)

iii) Find the transition matrix from B' to B. (3 Marks)

QUESTION THREE (20 MARKS)

a) Let $\mathbb{R}^3$ have the Euclidean inner product. Find the cosine of the angle between u and v given that $u = (-1, 5, 2), v = (2, 4, -9)$. (5 Marks)

b) Let $\mathbb{R}^3$ have the Euclidean inner product. Determine whether the following vectors form an orthonormal set

$u_1 = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$, $u_2 = (0,1,0)$, $u_3 = \left(\frac{-1}{\sqrt{2}}, 0, \frac{3}{\sqrt{2}}\right)$

(8 Marks)

c) Use the Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into an orthonormal basis using the Euclidean inner product in $\mathbb{R}^3$ where $u_1 = (1,1,1,)$ $u_2 = (-1,1,0), u_3 = (1,2,1)$. (7 Marks)

QUESTION FOUR (20 MARKS)

a) Find a matrix P that diagonalizes A, and hence determine $P^{-1}AP$, where $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ (10 Marks)

b) State the Cayley – Hamilton theorem and hence use the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$ to verify theorem. (10 Marks)

QUESTION FIVE (20 MARKS)

a) Find a matrix P that diagonalises the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

(10 Marks)

b) Use diagonalization to identify the conic section $2x^2 + 2y^2 - 4xy = 1$.

(10 Marks)