

Kasarani Campus
Off Thika Road
Tel. 2042692 / 3
P. O. Box 49274, 00100

NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175
KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR FIRST YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

Date: $13^{\text {th }}$ December, 2023
Time: 8.30am -10.30am

## KMA 201 - CALCULUS 1

## INSTRUCTIONS TO CANDIDATES

## ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

## QUESTION ONE (30 MARKS)

a) Evaluate the following integrals;
i) $\int \frac{3 x^{2}+\cos x}{\left(x^{3}+\sin x\right)^{10}} d x$ (3 marks)
ii) $\int x^{3} \sqrt{x^{4}-1} d x$ (2 marks)
iii) $\int \sin ^{3} 8 x \cos 8 x d x$ (2 marks)
iv) $\int x^{2} \cos x d x$
b) Find the curve whose slope at the point $(x, y)$ is $2 x+1$ if the curve is required to pass through the point $(1,-1)$.
(3 marks)
c) Define $R$ as the region bounded above by the graph of $f(x)=2 x-x 2$ and below by the $x$-axis over the interval [0,2]. Find the volume of the solid of revolution formed by revolving $R$ around the $y$-axis.
(4 marks)
d) Find the average value of $f(x)=6-2 x$ over the interval $[0,3]$.
(3 marks)
e) James and Kathy are racing on roller skates. They race along a long, straight track, and whoever has gone the farthest after 5 sec wins a prize. If James can skate at a velocity of $f(t)=5+2 t$ $\mathrm{ft} / \mathrm{sec}$ and Kathy can skate at a velocity of $g(t)=10+\cos \frac{\pi}{2} t \mathrm{ft} / \mathrm{sec}$, who is going to win the race?
f) Evaluate the following definite integrals
i) $\int_{0}^{1} x^{2}\left(1+2 x^{3}\right)^{5} d x$
(3 marks)
ii) $\quad \int_{0}^{1} e^{4 x^{2}+3} x d x$

## QUESTION TWO (20 MARKS)

a) Evaluate the following integrals:
i) $\int x \sqrt{2 x-1} \mathrm{dx}$ (4 marks)
ii) $\int \frac{1}{x(\ln x)^{8}} d x$ (3 marks)
iii) $\int x \cos x d x$ (4 marks)
iv) $\int e^{x} \cos x d x$ (3 marks)
v) $\int \cos ^{5} x d x$
b) Suppose a population of fruit flies increases at a rate of $g(t)=2 e^{0.02 t}$, in flies per day. If the initial population of fruit flies is 100 flies, how many flies are in the population after 10 days?
(3 marks)

## QUESTION THREE (20 MARKS)

a) Express the following functions in the form $P(x)+\frac{Q(x)}{R(x)}$
i) $f(x)=\frac{x^{4}+x}{x^{2}-3 x+2}$
ii) $f(x)=\frac{3 x^{2}-2 x-7}{x^{2}-x-2}$
b) Find the integral of $\int \frac{1}{x^{3}-2 x^{2}+x} d x$ using the method of partial fractions.
(6 marks)
c) If the motor on a motorboat is started at $t=0$ and the boat consumes gasoline at the rate of $5-t^{3}$ $\mathrm{gal} / \mathrm{hr}$, how much gasoline is used in the first 2 hours?
d) The parabola $y=x^{2}$ and the line $y=3 x+4$ enclose an area. Use integration methods to find the size of the area enclosed.

## QUESTION FOUR (20 MARKS)

a) Evaluate the following integrals:
i) $\int \frac{\sin t}{\cos ^{8} t} d t$
(3 marks)
ii) $\int \frac{x d x}{\sqrt{x-1}}$
(3 marks)
b) Evaluate the following definite integrals:
i) $\int_{0}^{\frac{\pi}{4}} \tan x \sec ^{2} x d x$
(4 marks)
ii) $\int_{0}^{\frac{\pi}{2}}(x+1) \sin x d x$
(3 marks)
iii) $\int_{0}^{1} x e^{-x} d x$
c) Find $\int_{0}^{6} y d x$ using Simpsons Rule given the following data

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 8 | 12 | 14 | 11 | 9 | 3 | 1 |

## QUESTION FIVE (20 MARKS)

a) Find the integral of the following using trigonometric substitution
i) $\quad \int \frac{d x}{x^{2} \sqrt{16-x^{2}}}$ (7 marks)
ii) $\int \frac{1}{x^{2} \sqrt{x^{2}+1}} d x$
(7 marks)
b) Use the slicing method to find the volume of the solid of revolution bounded by the graphs of $f(x)=x^{2}-4 x+5, x=1$, and $x=4$, and rotated about the $x$-axis.
(6 marks)

