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# KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR FIRST YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

Date: 13<sup>th</sup> December,2023 Time: 8.30am –10.30am

# KMA 201 - CALCULUS 1

### **INSTRUCTIONS TO CANDIDATES**

### ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

#### **QUESTION ONE (30 MARKS)**

			0 11			
a)	Evaluate	the	follow	ing	integr	als:
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i)	$\int \frac{3x^2 + \cos x}{(x^3 + \sin x)^{10}}  dx$	(3 marks)
ii)	$\int x^3 \sqrt{x^4 - 1}  dx$	(2 marks)
iii)	$\int \sin^3 8x \cos 8x  dx$	(2 marks)
iv)	$\int x^2 \cos x  dx$	(3 marks)

- b) Find the curve whose slope at the point (x, y) is 2x + 1 if the curve is required to pass through the point (1,-1). (3 marks)
- c) Define *R* as the region bounded above by the graph of  $f(x) = 2x x^2$  and below by the *x*-axis over the interval [0, 2]. Find the volume of the solid of revolution formed by revolving *R* around the *y*-axis. (4 marks)
- d) Find the average value of f(x) = 6 2x over the interval [0, 3].
- (3 marks) e) James and Kathy are racing on roller skates. They race along a long, straight track, and whoever has gone the farthest after 5 sec wins a prize. If James can skate at a velocity of f(t) = 5 + 2tft/sec and Kathy can skate at a velocity of  $g(t) = 10 + \cos \frac{\pi}{2} t ft/sec$ , who is going to win the race? (4 Marks)
- f) Evaluate the following definite integrals

i) 
$$\int_0^1 x^2 (1+2x^3)^5 dx$$
 (3 marks)  
ii)  $\int_0^1 e^{4x^2+3} x dx$  (3 marks)

### **QUESTION TWO (20 MARKS)**

- a) Evaluate the following integrals:
  - i)  $\int x \sqrt{2x 1} \, dx$  (4 marks) ii)  $\int \frac{1}{x(\ln x)^3} \, dx$  (3 marks)
  - iii)  $\int x \cos x dx$  (4 marks)

iv) 
$$\int e^x \cos x dx$$
 (3 marks)

v) 
$$\int \cos^5 x dx$$
 (3 marks)

b) Suppose a population of fruit flies increases at a rate of  $g(t) = 2e^{0.02t}$ , in flies per day. If the initial population of fruit flies is 100 flies, how many flies are in the population after 10 days? (3 marks)

#### **QUESTION THREE (20 MARKS)**

a) Express the following functions in the form  $P(x) + \frac{Q(x)}{R(x)}$ 

i) 
$$f(x) = \frac{x^4 + x}{x^2 - 3x + 2}$$
 (3 Marks)  
ii)  $f(x) = \frac{3x^2 - 2x - 7}{x^2 - x - 2}$  (3 marks)

b) Find the integral of  $\int \frac{1}{x^3 - 2x^2 + x} dx$  using the method of partial fractions.

(6 marks)

- c) If the motor on a motorboat is started at t = 0 and the boat consumes gasoline at the rate of 5 − t<sup>3</sup> gal/hr, how much gasoline is used in the first 2 hours? (4 marks)
   d) The parabola y = x<sup>2</sup> and the liney = 3x + 4 enclose an area. Use integration methods to find the
  - size of the area enclosed. (6 marks)

#### **QUESTION FOUR (20 MARKS)**

- a) Evaluate the following integrals:
  - i)  $\int \frac{\sin t}{\cos^3 t} dt$  (3 marks) ii)  $\int \frac{x \, dx}{\sqrt{x-1}}$  (3 marks)
- b) Evaluate the following definite integrals:  $\pi$

i) 
$$\int_0^4 \tan x \sec^2 x dx$$

π

(4 marks)

ii) 
$$\int_{0}^{2} (x+1)\sin x dx$$
 (3 marks)  
iii)  $\int_{0}^{1} x e^{-x} dx$  (3 marks)

c) Find  $\int_0^6 y dx$  using Simpsons Rule given the following data

Х	0	1	2	3	4	5	6
у	8	12	14	11	9	3	1

(4 marks)

## **QUESTION FIVE (20 MARKS)**

a) Find the integral of the following using trigonometric substitution

i) 
$$\int \frac{dx}{x^2\sqrt{16-x^2}}$$
 (7 marks)  
ii) 
$$\int \frac{1}{x^2\sqrt{x^2+1}} dx$$
 (7 marks)

b) Use the slicing method to find the volume of the solid of revolution bounded by the graphs of 
$$f(x) = x^2 - 4x + 5$$
,  $x = 1$ , and  $x = 4$ , and rotated about the x-axis.

(6 marks)