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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
SECOND YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF BUSINESS AND INFORMATION TECHNOLOGY

Date: 9th April, 2024
Time: 8.30am –10.30am

KMA 2213 - MATHEMATICAL STATISTICS

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) Given the following data $E(X)=12.3$, $E(Y)=8.5$, $Var(X)=4.45$, $Var(Y)=5.02$, and that $Cov(X,Y)=-3.15$. Find

i) $E(2X-1)$, (2 Marks)

ii) $Var(4Y+3X)$ (3 Marks)

b) Under what circumstances do a researcher prefer sampling to complete enumeration. (3 Marks)

c) The weight of a new pack of an item is known to have a mean of 40 grams. A sample of 7 packages yielded the following weights: 37, 38, 39, 40, 38, 41 and 42 grams. Obtain point estimate of population mean μ and population variance σ^2 (5 Marks)

d) A company produces jars of English Honey. The weight of the glass jars used are normally distributed with a mean 122.3g and standard deviation of 2.6g. calculate the probability that a randomly chosen jar will weigh

i) Less than 127g (3 Marks)

ii) Less than 129.2g but more than 124.5g (4 Marks)

e) The probability that an hourly employee will refuse to work overtime (allowed by the union contract) is 0.30. Let X be the number of employees refusing to work overtime. If ten are asked, what is the probability that not less than five will work overtime? (5 Marks)

f) A population having 400 sampling units is known to have a mean, $\bar{Y}=25$ and variance $S^2=55.6$. In SRSWOR, how many sampling units should be chosen to estimate $\bar{Y}$ with a permissible error of 5% and at 98% confidence coefficient (5 Marks)

QUESTION TWO (20 MARKS)

a) Given that

$$f(x,y)=\begin{cases}\frac{1}{6}(x+4y), & 0\le x\le 2,\ 0\le y\le 1\\ 0, & elsewhere\end{cases},$$

Obtain

i) $E(X)$ (2 Marks)

ii) $E(Y)$ (2 Marks)

b) Given that X is a normal random variable describing the distribution patterns of a mental condition with mean 20 and variance 16, compute $\Pr(|X-20|<3.2)$ (3 Marks)

c) The following data is a simple random sample without replacement from a population of size 100: 5, 8, 4, 7 and 6. Pointing out the assumptions you make,

i) Estimate the population mean (2 Marks)

ii) Estimate variance of the estimator in (i) above (3 Marks)

iii) Construct the 95% confidence interval for the population mean $\overline{Y}$ (4 Marks)

d) The daily demand (X = number of orders per day) for cheesecake at Friday's Restaurant is approximately normally distributed with a standard deviation 2.8. Find the mean number of orders if less than 20 orders are received only 4% of the time (4 Marks)

QUESTION THREE (20 MARKS)

a) A random sample of size 15 from a normal population with mean μ and variance σ^2 yields $\sum_{i=1}^{15} x_i = 48$ and $\sum_{i=1}^{15} x_i^2 = 271.2$, obtain the unbiased point estimate for population standard deviation σ (4 Marks)

b) A random sample of 31 observations from a normal population possessed a mean $\overline{X}=90$ and a sample standard deviation of $s=7.1$. test $H_0:\sigma^2=40$ versus $H_1:\sigma^2>40$ (5 Marks)

c) An experiment aimed at assessing the correlation between two variables X and Y, the following data was produced

X	13.0	16.0	19.0	22.0	25.0	28.0
Y	36.5	37.8	40.2	39.3	41.7	43.0

i) Calculate the arithmetic mean values of X and Y (2 Marks)

ii) Obtain the equation of the regression line (5 Marks)

iii) Compute correlation coefficient and use the obtained value to comment on the nature of correlation between the two variables (4 Marks)

QUESTION FOUR (20 MARKS)

a) Suppose that X and Y have a bivariate distribution given by

$$f(x,y)=\begin{cases} p^{x+y}(1-p)^{2-x-y}, & x=0,1 \text{ and } y=0,1 \\ 0, \quad Otherwise \end{cases}$$

Obtain the following;

i) Variance of X (3 Marks)

ii) Variance of Y (3 Marks)

b) The tensile strength of a certain metal component is normally distributed with a mean of 10,000 kilograms per square centimeter and a standard deviation of 100 kilograms per square centimeter. Measurements are recorded to the nearest 50 kilograms per square centimeter.

i) What proportion of these components exceed 10,150 kilograms per square centimeter in tensile strength? (3 Marks)

ii) If specifications require that all components have tensile strength between 9800 and 10,200 kilograms per square centimeter inclusive, what proportion of pieces would we expect to meet this specification? (3 Marks)

c) A firm conducts a study to determine if absenteeism of day workers is different from those of night shifts. 150 workers are taken from each shift. The result show that 37 day workers have been absent at least 5 times over the past one year, while 52 night workers have missed at least 5 times. Use 90% confidence interval for the difference in proportion (4 Marks)

d) In studying the association between two variables X and Y, a researcher obtained the following data:

$\sum x_i = 25.7 \quad \sum y_i = 14.4 \quad \sum x_i^2 = 88.31 \quad \sum x_i y_i = 46.856 \quad \sum y_i^2 = 26.4324 \quad n = 8$

Determine the value of the coefficient of correlation between the two variables and comment in the value obtained (4 Marks)

QUESTION FIVE(20 MARKS)

a) Suppose a random variable X has a normal distribution with mean μ and variance σ^2. Derive the distribution of the standard normal variate Z defined as $Z=\frac{X-\mu}{\sigma}$ (5 Marks)

b) In a consignment of 20 articles, just 4 are defective. If a random sample of 5 articles is taken from this consignment,

i) Give the probability distribution describing this scenario (2 Marks)

ii) Find the probability that the sample will contain no defective article (3 Marks)

iii) Find the probability that the sample will contain 3 or more defective articles. (4 Marks)

c) Test the hypothesis that the average content of containers of a particular lubricant is 10 litres against not equal if the contents of a random sample of 10 containers are 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3 and 9.8 litres. Use 0.01 level of significance and assume a normal distribution (6 Marks)