KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
FIRST YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF BUSINESS AND INFORMATION TECHHOLOGY
KMA 2102 - CALCULUS FOR BUSINESS INFORMATION

Date: 8ᵗʰ August, 2023
Time: 11.30 am – 1.30pm

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS
QUESTION ONE (30 MARKS)

a)   Find domain and range for each of the following functions;

   i) $f(x)=(x-4)^2+5$ (2 Marks)

   ii) $f(x)=\sqrt{3x+2}-1$ (2 Marks)

   iii) $f(x)=\dfrac{3}{x-2}$ (2 Marks)

b)   Evaluate the limits;

   i) $\lim\limits_{x\to-1}\dfrac{\sqrt{x+2}-1}{x+1}$ (2 Marks)

   ii) $\lim\limits_{x\to3}\dfrac{x^2-3x}{2x^2-5x-3}$ (2 Marks)

c)   Find the derivative of the function $f(x)=x^2-2x$ using first principle method. (3 Marks)

d)   The position of a particle along a straight line at time t seconds is given by

   $s(t)=3t^2-4t+1$. Find the function that describes its acceleration at time t. (3 Marks)

e)   Given $(x)=x^2+1\wedge g(x)=\dfrac{1}{x}$, find each of the following:

   i) $(foh)(x)$ (2 Marks)

   ii) $(gof)(x)$ (2 Marks)

f)   Differentiate the following functions:

   i) $e^x(2x^2+1)$ (2 Marks)

   ii) $\ln(x^2+2)$ (2 Marks)

   iii) $\cos^3(4\theta+1)$ (2 Marks)

   iv) $e^{3x}\sin(2x+1)$ (2 Marks)

   v) $y=\left(\dfrac{x}{3x+2}\right)^5$ (2 Marks)

QUESTION TWO (20 MARKS)

a) Find the equation of the tangent to the curve with equation $y=\sin x$ at the point $x=\dfrac{\pi}{3}$.

(4 Marks)

b) Find the second derivatives of the following functions;

    i) $x^2+y^2=25$ (3 Marks)

    ii) $y=\ln\left(1+t^4\right)$ (3 Marks)

c) Find the x-coordinates of the points of inflection for the curve with equation;

$$y=\frac{x^4}{12}+\frac{x^3}{3}-\frac{3x^2}{2}+3x-7$$

(4 Marks)

d) Verify that the following functions satisfy the criteria stated in Rolle 's Theorem and find all values c in the given interval where $f'(c)=0$

    i) $f(x)=x^2+2x$ over $[-2,0]$ (3 Marks)

    ii) $f(x)=x^3-4x$ over $[-2,2]$ (3 Marks)

QUESTION THREE (20 MARKS)

a) Find the equation of the line tangent to the graph $f(x)=x^2-4x+6$ at $x=1$.   (3 Marks)

b) Differentiate the following function implicitly;

    i) $y^2+x^3-y^3+6=3y$ (3 Marks)

    ii) $y^2+x^3-xy+\cos y=0$ (3 Marks)

c) A particle moves along a coordinate axis in such a way its position that at time t is given by $s(t)=2\sin t-t$ for $0\le t\le 2\pi$. At what time is the particle at rest?   (3 Marks)

d) Find the derivatives of the following;

    i) $y=\tan\left(4x^2-3x+1\right)$ (2 Marks)

    ii) $y=5x^3\sin x$ (2 Marks)

    iii) $f(x)=\dfrac{\cos x}{4x^2}$ (2 Marks)

    iv) $y=\cos^4\left(7x^2+1\right)$ (2 Marks)

QUESTION FOUR (20 MARKS)

a) Evaluate the limits;

    i) $\lim\limits_{x\to 0}\dfrac{e^x-1}{x}$ (3 Marks)

    ii) $\lim\limits_{x\to\infty}\dfrac{3x^5+x^3-5}{6+x+7x^2}$ (3 Marks)

b) Show that $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$ (4 Marks)

c) A rocket is launched so that it rises vertically. A camera is positioned 5000 feet from the launch pad. When the rocket is 100 feet above the launch pad, its velocity is 600 ft/sec. Find the rate of change of the camera's angle as a function of time so that it stays focused on the rocket.

(10 Marks)

QUESTION FIVE (20 MARKS)

a) Find the extrema of the following function $f(x)=-x^2+3x-2$ over $[1,3]$.   (7 Marks)

b) A ball is dropped from a height of 64 feet. Its height above the ground (in feet) t seconds later is given by $s(t)=-16t^2+64$.

i)   What is the instantaneous velocity of the ball when it hits the ground?   (3 Marks)

ii)   What is the average velocity during its fall.   (3 Marks)

c)   A rocket travels in an elliptical orbit whose path is given by the equation $45\,x^2+25\,y^2=100$. The rocket can fire missiles along lines tangent to its path. If the rocket fires a missile when it is located at $\left(3,\dfrac{8}{3}\right)$, where will it intersect the x-axis?   (7 Marks)